

# STRUCTURAL, MODAL, AND PRELIMINARY AEROELASTIC DESIGN STUDY OF A ROCKET FIN

Finite-Element Modal Analysis • Static Aerodynamic Loading • Parametric Trade Studies

## **Mechanical Engineering Portfolio Project**

SOLIDWORKS Simulation • Microsoft Excel • Analytical Validation

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## Executive Summary

This report presents a complete structural and preliminary aeroelastic design study of a trapezoidal rocket fin. A baseline CAD and modal model was established in SOLIDWORKS Simulation, independently checked with a simplified analytical cantilever calculation, and verified through mesh refinement. The validated model was then used for parametric studies of thickness, span, material, root-support stiffness, and constant-area sweep. The sweep study was repeated with planform area and mass held fixed so that the structural effect of tip-chord offset could be isolated.

The study then evaluated structural response under two levels of simplified aerodynamic loading. The original  $p = q$  case was retained as a conservative unit-coefficient reference load, while a new coefficient-based finite-wing screening model tied pressure to angle of attack. At 100 m/s, the unit-coefficient reference produced 31.50 MPa maximum von Mises stress and 1.716 mm maximum displacement. A 5° angle-of-attack screening case produced an equivalent pressure of 960 Pa, 4.94 MPa maximum stress, and 0.269 mm maximum displacement. The finite-element response matched the expected linear pressure scaling essentially exactly.

Finally, the report introduces a preliminary aeroelastic screening framework using bending/torsional modal separation and reduced frequency. No true flutter speed is claimed because the project did not include coupled unsteady aerodynamic stiffness and damping or a formal flutter eigenvalue solution.

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# 1. Introduction

## 1.1 Engineering Motivation

Rocket fins are lightweight cantilevered structures that must provide aerodynamic stability without becoming excessively flexible or dynamically vulnerable. Their structural response depends on geometry, material stiffness and density, attachment conditions, and aerodynamic loading. A fin that is too flexible can experience large static deflections, reduced modal frequencies, and stronger interaction between structural vibration and aerodynamic forces.

The project therefore focuses on structural dynamics first, then progressively introduces load and aeroelastic screening. This sequencing is important: a credible flutter investigation requires a trustworthy structural model before aerodynamic coupling can be meaningfully interpreted.

## 1.2 Project Objectives

- Create a repeatable baseline rocket-fin CAD model.
- Identify the first bending and torsional vibration modes using finite-element modal analysis.
- Validate the first bending frequency using a simplified analytical estimate.
- Demonstrate mesh convergence of the baseline modal solution.
- Quantify sensitivity to thickness, span, sweep parameter, material, and root stiffness.
- Evaluate static stress and displacement under simplified aerodynamic pressure loading.
- Compare structural performance across multiple flight speeds.
- Develop a complete thickness trade study combining mass, modal response, stress, and displacement.
- Introduce reduced frequency and modal separation as preliminary aeroelastic screening quantities.
- Select a balanced baseline configuration and clearly document what the model can and cannot claim.

# 2. Baseline Fin Model and Finite-Element Setup

## 2.1 Geometry

| Geometry parameter | Value  |
|--------------------|--------|
| Root chord         | 200 mm |
| Tip chord          | 100 mm |
| Span               | 150 mm |
| Sweep parameter    | 50 mm  |
| Thickness          | 3 mm   |

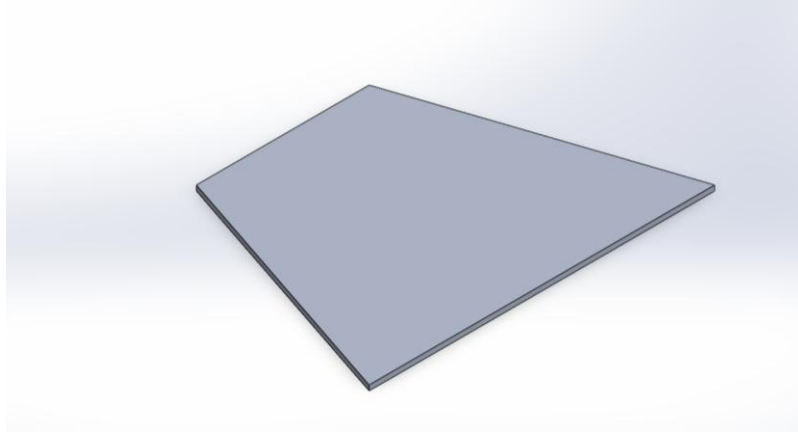


Figure 1. Baseline 3 mm trapezoidal rocket fin geometry modeled in SOLIDWORKS.

## 2.2 Material and Mass

| Property                                 | Value                  |
|------------------------------------------|------------------------|
| Material                                 | 6061-T6 aluminum       |
| Baseline mass                            | 182.25 g               |
| Elastic modulus used in analytical check | 69 GPa                 |
| Density used in analytical check         | 2700 kg/m <sup>3</sup> |

## 2.3 Boundary Condition

The root face was modeled as fully fixed in the baseline finite-element studies. All translational and rotational motion at that face was constrained. This idealized condition provides a repeatable upper-stiffness reference for comparing design changes.

## 2.4 Mesh

The baseline model was meshed using SOLIDWORKS Simulation solid elements. A mesh-refinement check was later performed to determine whether the first two modal frequencies were sensitive to the chosen mesh density.

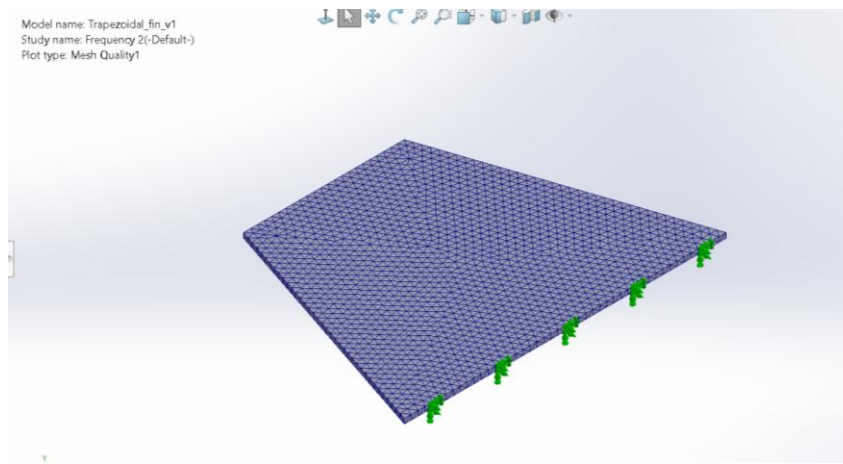


Figure 2. Fully fixed root boundary condition and Baseline finite-element mesh.

### 3. Phase 1 — Baseline Structural and Modal Characterization

#### 3.1 Analytical First-Bending Frequency Estimate

To provide an independent check of the finite-element model, the fin was approximated as a uniform rectangular cantilever beam. Because the true planform is trapezoidal, the mean chord was used as the equivalent beam width.  $b = (200 + 100)/2 = 150\text{mm} = 0.150\text{m}$

The analytical values were:

- Span  $L = 0.150\text{ m}$
- Equivalent width  $b = 0.150\text{ m}$
- Thickness  $t = 0.003\text{ m}$
- Elastic modulus  $E = 69 \times 10^9\text{ Pa}$
- Density  $\rho = 2700\text{ kg/m}^3$

$$A = bt = (0.150)(0.003) = 4.50 \times 10^{-4}\text{m}^2$$

$$I = bt^3/12 = 3.375 \times 10^{-10}\text{m}^4$$

$$f_1 = [1.875^2/(2\pi L^2)]\sqrt{(EI/\rho A)} = 108.87\text{Hz}$$

The analytical result was compared with the SOLIDWORKS first bending frequency of 136.19 Hz. The analytical model underpredicted the finite-element value by approximately 20.06%. The difference is expected because the real fin is a swept trapezoidal plate with a nonuniform planform, while the hand calculation treats it as a uniform beam. The calculation is therefore used as an approximate validation rather than an exact prediction.

#### 3.2 Baseline Modal Behavior

| Mode   | Interpretation | Natural frequency |
|--------|----------------|-------------------|
| Mode 1 | First bending  | 136.19 Hz         |
| Mode 2 | First torsion  | 381.54 Hz         |

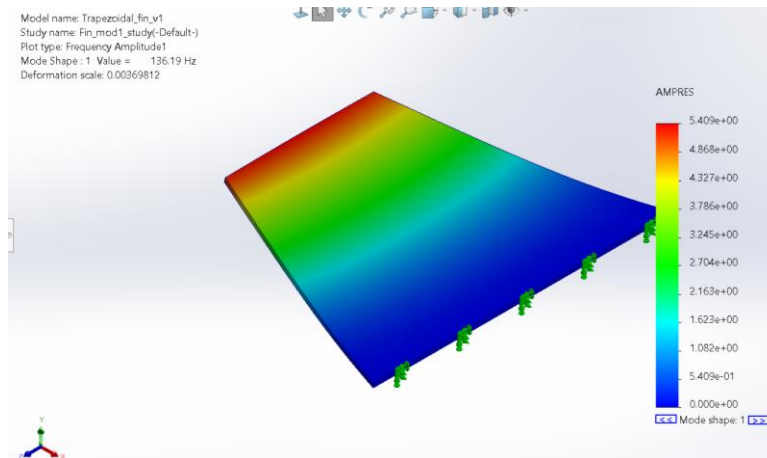


Figure 3. Mode 1 — first bending at 136.19 Hz

The first mode was dominated by bending away from the undeformed fin plane. The root remained fixed and the largest modal motion occurred toward the free region.

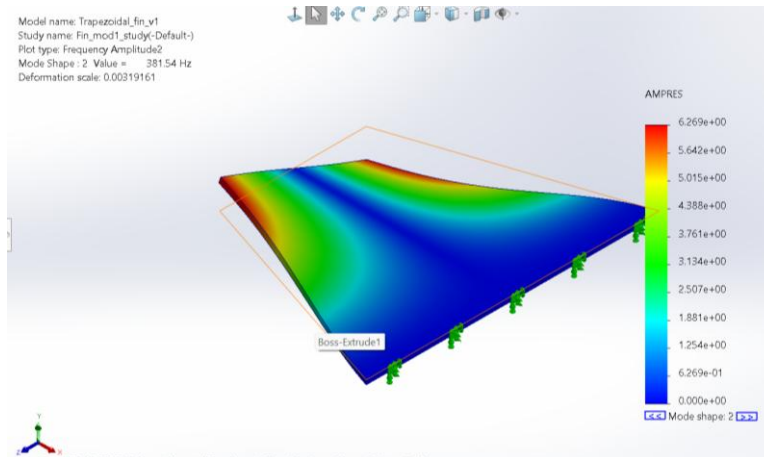


Figure 4. Mode 2 — first torsion at 381.54 Hz

The second mode showed opposite deformation across different portions of the free region, identifying the mode as primarily torsional. The bending and torsional modes are the most relevant low-order structural modes for later aeroelastic interpretation.

### 3.3 Mesh-Convergence Check

| Mode          | Original mesh | Finer mesh | Change  |
|---------------|---------------|------------|---------|
| First bending | 136.19 Hz     | 136.19 Hz  | ≈0.000% |
| First torsion | 381.56 Hz     | 381.54 Hz  | ≈0.005% |

The first bending frequency showed no meaningful change with mesh refinement. The torsional frequency changed by approximately 0.005%. The modal solution was therefore considered sufficiently mesh-converged for the sensitivity studies.

### 3.4 Fin-Thickness Sensitivity Study

Three geometrically identical aluminum fins were analyzed at 2 mm, 3 mm, and 4 mm thickness. Material, planform geometry, root fixture, and analysis procedure were kept constant.

| Thickness | Mass     | First bending | First torsion |
|-----------|----------|---------------|---------------|
| 2 mm      | 121.50 g | 90.83 Hz      | 254.97 Hz     |
| 3 mm      | 182.25 g | 136.19 Hz     | 381.54 Hz     |
| 4 mm      | 243.00 g | 181.51 Hz     | 507.41 Hz     |

From 2 to 3 mm, the first bending frequency increased by approximately 49.94% and the first torsional frequency increased by approximately 49.64%. From 3 to 4 mm, the increases were approximately 33.28% and 32.99%, respectively.

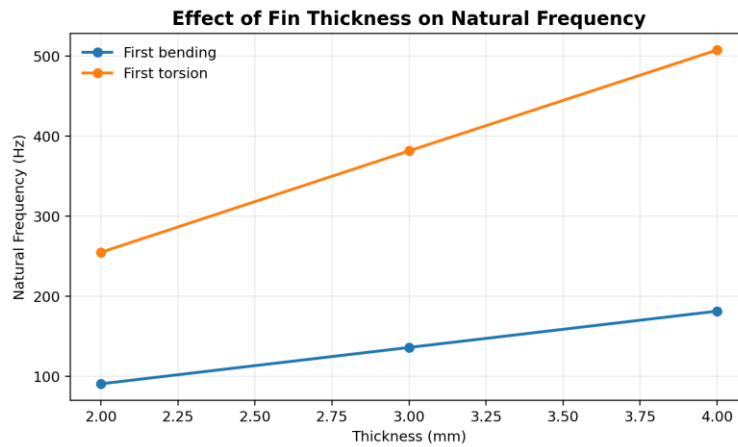


Figure 5. Effect of fin thickness on the first bending and torsional natural frequencies.

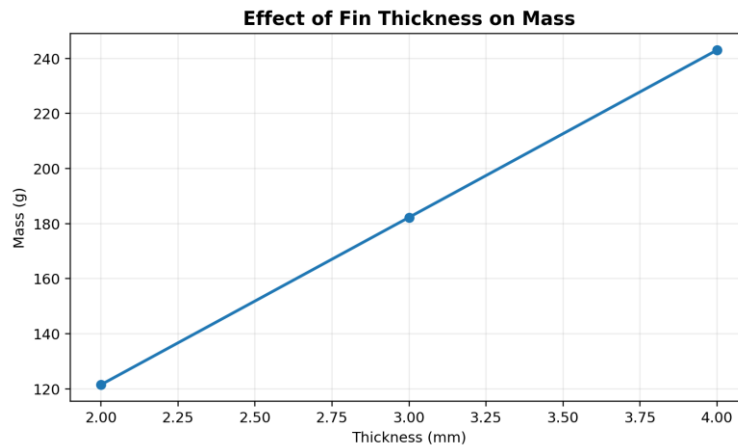


Figure 6. Effect of fin thickness on fin mass.

### 3.5 Frequency-to-Mass Tradeoff

| Thickness | Bending frequency / mass | Torsional frequency / mass |
|-----------|--------------------------|----------------------------|
| 2 mm      | 0.748 Hz/g               | 2.099 Hz/g                 |
| 3 mm      | 0.747 Hz/g               | 2.093 Hz/g                 |
| 4 mm      | 0.747 Hz/g               | 2.088 Hz/g                 |

The frequency-to-mass ratios remained almost constant. Increasing aluminum thickness therefore increases absolute stiffness and natural frequency, but it does not produce a major gain in modal performance per unit mass.

### 3.6 Phase 1 Engineering Conclusions

- The 3 mm baseline fin has a first bending frequency of 136.19 Hz and first torsional frequency of 381.54 Hz.
- Mesh refinement produced negligible changes in the first two modal frequencies.
- The simplified cantilever estimate predicted 108.87 Hz, approximately 20.06% below the finite-element result.
- Increasing thickness strongly increased bending and torsional frequencies.
- Mass increased almost proportionally with thickness and modal frequency.

- Frequency-to-mass performance remained nearly constant, motivating later geometry and material studies.

## 4. Phase 2 — Geometry Sensitivity Study

Phase 2 investigated the influence of fin span and sweep parameter while retaining the baseline material, 3 mm thickness, root chord, tip chord, fixed-root boundary condition, and modal-analysis procedure.

### 4.1 Fin-Span Sensitivity

| Span   | Mass     | First bending | First torsion |
|--------|----------|---------------|---------------|
| 120 mm | 145.80 g | 211.36 Hz     | 510.22 Hz     |
| 150 mm | 182.25 g | 136.19 Hz     | 381.54 Hz     |
| 180 mm | 218.70 g | 94.88 Hz      | 301.94 Hz     |

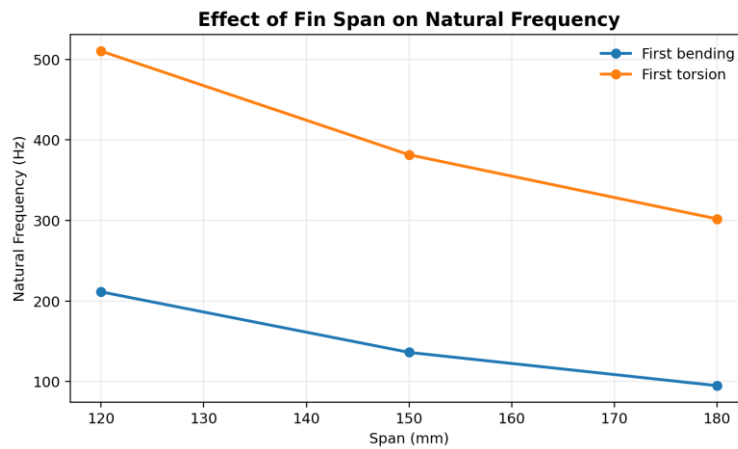


Figure 7. Effect of fin span on the first bending and torsional natural frequencies.

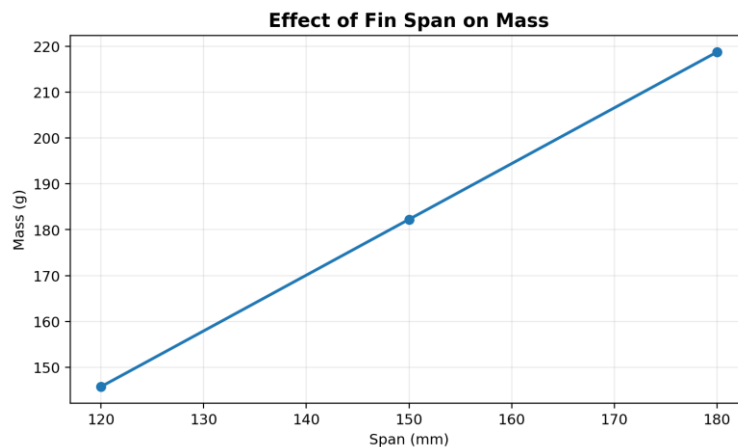


Figure 8. Effect of fin span on mass.

Increasing span produced a substantial reduction in both natural frequencies while increasing mass. The first bending frequency decreased from 211.36 Hz at 120 mm span to 94.88 Hz at 180 mm. The first torsional frequency decreased from 510.22 Hz to 301.94 Hz over the same range. The longer unsupported distance makes the fin structurally more flexible, so span is a particularly influential geometric parameter.

## 4.2 Constant-Area Fin-Sweep Sensitivity

| Tip-chord offset | Mass     | First bending | First torsion |
|------------------|----------|---------------|---------------|
| 25 mm            | 182.25 g | 134.91 Hz     | 378.18 Hz     |
| 50 mm            | 182.25 g | 136.19 Hz     | 381.54 Hz     |
| 75 mm            | 182.25 g | 134.91 Hz     | 378.17 Hz     |

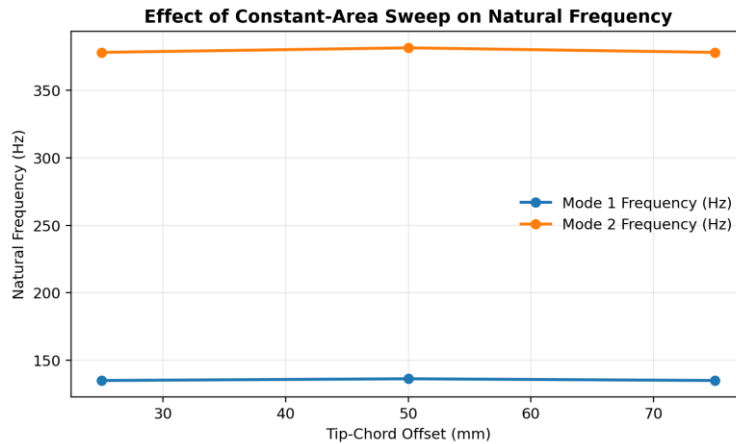


Figure 9. Effect of constant-area tip-chord offset on the first bending and torsional natural frequencies.

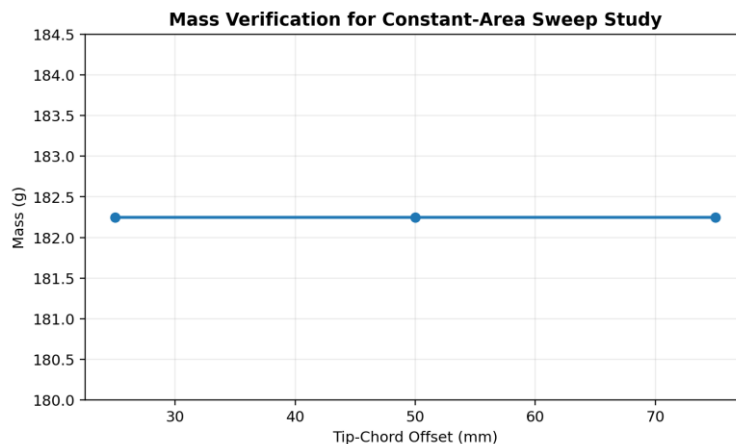


Figure 10. . Mass verification for the constant-area sweep study.

The sweep study was repeated using a constant-area geometry in which the 100 mm tip chord was translated relative to the 200 mm root chord while root chord, tip chord, span, thickness, material, planform area, and mass were held fixed. All three cases retained a planform area of 22,500 mm<sup>2</sup> and a mass of 182.25 g.

With area and mass isolated, sweep alone had only a minor effect on the first two natural frequencies. Relative to the 50 mm baseline, the 25 mm and 75 mm offsets reduced Mode 1 by about 0.94% and Mode 2 by about 0.88%. The large changes observed in the original sweep study were therefore mainly associated with simultaneous changes in planform area and mass distribution rather than sweep alone.

## 4.3 Comparison of Geometry Effects

- Increasing span increased mass and substantially reduced bending and torsional frequencies.

- When planform area and mass were held constant, changing the tip-chord offset from 25 to 75 mm altered the first two modal frequencies by less than 1% relative to the 50 mm baseline.
- Span strongly influences bending stiffness because it changes the unsupported length.
- The corrected constant-area sweep study shows that isolated sweep is a secondary structural variable for the first two modes over the tested range.
- Geometry modification can produce more favorable mass–stiffness changes than simply increasing thickness.

#### 4.4 Phase 2 Engineering Conclusions

The geometry study demonstrated that span is a major structural variable, while isolated sweep has only a small effect on the first two modal frequencies for this fin. The corrected constant-area sweep cases also show why controlled parameterization is necessary: the strong trend in the original sweep study was largely caused by accompanying area and mass changes.

### 5. Phase 3 — Material Sensitivity Study

#### 5.1 Study Setup

The built-in glass-fiber and carbon-cloth library entries available during the project did not contain a complete set of elastic properties required for the modal study. Custom equivalent isotropic materials were therefore created for preliminary comparison.

| Material                 | Elastic modulus                 | Poisson ratio | Shear modulus | Density                |
|--------------------------|---------------------------------|---------------|---------------|------------------------|
| 6061-T6 aluminum         | 69 GPa used in analytical check | —             | —             | 2700 kg/m <sup>3</sup> |
| E-Glass/Epoxy equivalent | 25 GPa                          | 0.25          | 10 GPa        | 1900 kg/m <sup>3</sup> |
| Carbon/Epoxy equivalent  | 60 GPa                          | 0.30          | 23.08 GPa     | 1600 kg/m <sup>3</sup> |

The composite properties are screening assumptions rather than laminate-certified data. Real composite fins are anisotropic and require orthotropic lamina definitions, stacking sequence, ply orientations, and composite failure criteria.

#### 5.2 Material Study Results

| Material                 | Mass     | Mode 1    | Mode 2    | Mode 1 / Mass | Mode 2 / Mass |
|--------------------------|----------|-----------|-----------|---------------|---------------|
| 6061-T6 Aluminum         | 182.25 g | 136.19 Hz | 381.54 Hz | 0.747 Hz/g    | 2.093 Hz/g    |
| E-Glass/Epoxy Equivalent | 128.25 g | 96.10 Hz  | 276.86 Hz | 0.749 Hz/g    | 2.159 Hz/g    |
| Carbon/Epoxy Equivalent  | 108.00 g | 163.80 Hz | 463.90 Hz | 1.517 Hz/g    | 4.295 Hz/g    |

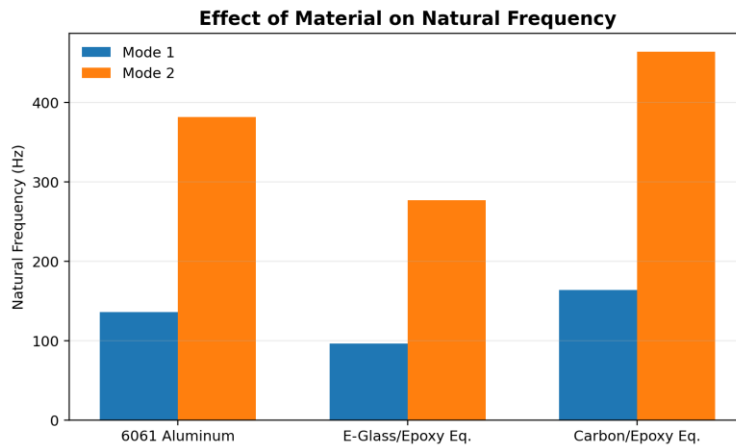


Figure 11. Effect of material model on the first two natural frequencies.

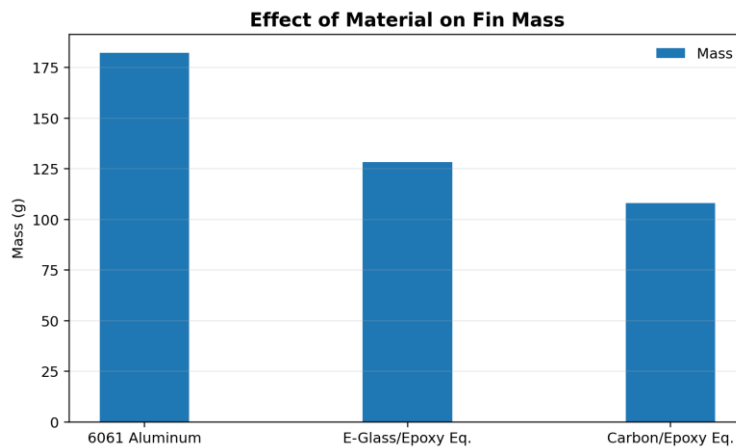


Figure 12. Effect of material model on fin mass.

The E-glass/epoxy equivalent substantially reduced mass but also reduced both modal frequencies. The carbon/epoxy equivalent produced the lowest mass while simultaneously producing the highest Mode 1 and Mode 2 frequencies. Under the simplified isotropic assumptions, carbon/epoxy therefore provided the strongest stiffness-to-mass performance.

### 5.3 Phase 3 Engineering Conclusions

- Reducing density alone does not guarantee improved dynamic performance.
- E-glass/epoxy reduced mass but also reduced modal frequencies.
- Carbon/epoxy reduced mass by approximately 41% relative to aluminum.
- Carbon/epoxy increased the first bending frequency to 163.80 Hz and first torsional frequency to 463.90 Hz.
- Carbon/epoxy showed the best frequency-to-mass performance in the simplified screening model.
- A proper composite design still requires an orthotropic laminate model and verified material data.

## 6. Phase 4 — Root-Stiffness Sensitivity Study

### 6.1 Motivation

The baseline fully fixed root is an idealization. A real fin attachment can be compliant because of the fin tab, adhesive, fillet, airframe, and local structure. Phase 4 therefore investigated how sensitive the modal solution is to root support stiffness.

### 6.2 Elastic-Support Cases

The same baseline 3 mm aluminum geometry was retained, but the fixed fixture was replaced with an elastic support on the root face. Total normal and shear stiffness were varied together.

| Normal stiffness       | Shear stiffness        |
|------------------------|------------------------|
| $1 \times 10^6$ N/m    | $1 \times 10^6$ N/m    |
| $1 \times 10^7$ N/m    | $1 \times 10^7$ N/m    |
| $1 \times 10^8$ N/m    | $1 \times 10^8$ N/m    |
| $1 \times 10^9$ N/m    | $1 \times 10^9$ N/m    |
| $1 \times 10^{10}$ N/m | $1 \times 10^{10}$ N/m |

These stiffness values were not measured physical joint properties; they were used as a parametric sensitivity range.

### 6.3 Root-Stiffness Results

| Root stiffness         | Lowest system natural frequency |
|------------------------|---------------------------------|
| $1 \times 10^6$ N/m    | 4.08 Hz                         |
| $1 \times 10^7$ N/m    | 12.85 Hz                        |
| $1 \times 10^8$ N/m    | 39.09 Hz                        |
| $1 \times 10^9$ N/m    | 93.41 Hz                        |
| $1 \times 10^{10}$ N/m | 128.64 Hz                       |
| Fully fixed reference  | 136.19 Hz                       |

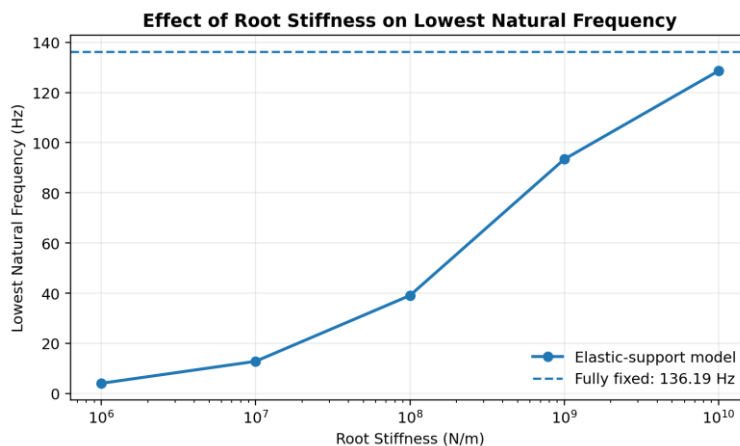


Figure 13. Effect of root stiffness on the lowest system natural frequency, extended to  $1 \times 10^{10}$  N/m.

The lowest system natural frequency increased strongly with support stiffness, rising from 4.08 Hz at  $1 \times 10^6$  N/m to 128.64 Hz at  $1 \times 10^{10}$  N/m. The highest elastic-support case was only about 5.5% below the fully fixed reference of 136.19 Hz.  $f \propto \sqrt{k}$

## 6.4 First-Order Attachment Stiffness Estimate

To ground the sensitivity study in a physical attachment estimate, the adhesive bond was approximated as a shear spring using  $k_s = GA/t_a$ . A 200 mm bond length, 25 mm effective bond width, 0.5 mm adhesive thickness, and 1.0 GPa adhesive shear modulus give a bonded area of 0.005 m<sup>2</sup> and an estimated shear stiffness of  $1.0 \times 10^{10}$  N/m.

$$k_s = GA/t_a = (1.0 \times 10^9)(0.005)/(0.0005) = 1.0 \times 10^{10} \text{ N/m}$$

| Parameter                 | Value                    |
|---------------------------|--------------------------|
| Bond length               | 0.200 m                  |
| Effective bond width      | 0.025 m                  |
| Bonded area               | 0.005 m <sup>2</sup>     |
| Adhesive thickness        | 0.0005 m                 |
| Adhesive shear modulus    | $1.0 \times 10^9$ Pa     |
| Estimated shear stiffness | $1.0 \times 10^{10}$ N/m |

This estimate is intentionally first-order and does not include fin-tab bending, fillets, airframe-shell compliance, adhesive nonlinearities, or rotational joint stiffness. Nevertheless, the  $1 \times 10^{10}$  N/m SOLIDWORKS case produced 128.64 Hz, suggesting that a sufficiently stiff bonded attachment can approach the fully fixed modal limit.

## 6.5 Mode-Shape Behavior

Introducing elastic root compliance also changed mode ordering and mode character. Additional low-frequency support-coupled motion appeared, so a mode number could no longer be assumed to represent the same physical deformation pattern across all boundary conditions.

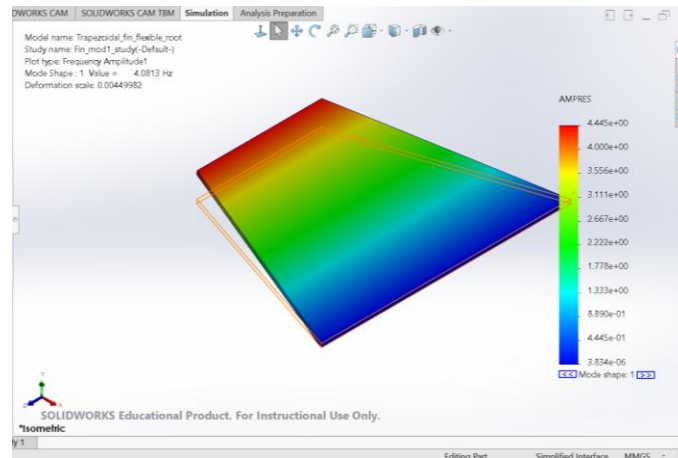


Figure 14. Representative low-frequency mode shape of the fin with elastic root support, showing support-coupled deformation at 39.09 Hz.

For rigorous future tracking, mode shapes should be compared directly or correlated using a Modal Assurance Criterion rather than being identified only by mode number.

## 6.6 Phase 4 Engineering Conclusions

- The fully fixed root provides the stiffest idealized boundary condition.
- Root compliance introduced low-frequency support-dominated motion, while the physically motivated high-stiffness estimate approached the fully fixed response.

- Increasing elastic-support stiffness substantially increased the lowest system natural frequency.
- Even the  $10^9$  N/m case remained below the fully fixed reference.
- Boundary-condition assumptions can materially affect aeroelastic conclusions and should be refined in future work.

## 7. Phase 5 — Static Aerodynamic Loading

### 7.1 Objective and Two-Level Load Model

Phase 5 evaluated the structural response of the baseline 3 mm 6061-T6 aluminum fin using two related load models. First, the original uniform pressure  $p = q$  was retained as a conservative unit-coefficient reference case for direct comparison across thickness and speed studies. Second, a coefficient-based finite-wing screening model was introduced to connect the equivalent normal pressure to angle of attack rather than treating dynamic pressure itself as the flight load.

$$q = \frac{1}{2}\rho V^2$$

The density used was  $\rho = 1.225 \text{ kg/m}^3$ . The  $p = q$  model is therefore interpreted only as a controlled unit-coefficient reference load, not as a prediction of actual fin pressure. The coefficient-based model below provides a more physically motivated first-order estimate while still remaining simpler than CFD or a full rocket-fin aerodynamic model.

### 7.2 Unit-Coefficient Reference Case — 100 m/s

$$q = 1/2(1.225)(100^2) = 6125 \text{ Pa}$$

| Result                              | 100 m/s unit-coefficient reference |
|-------------------------------------|------------------------------------|
| Maximum von Mises stress            | 31.50 MPa                          |
| Maximum resultant displacement      | 1.716 mm                           |
| Yield strength — 6061-T6 Aluminum   | 276 MPa                            |
| Approximate static factor of safety | 8.76                               |

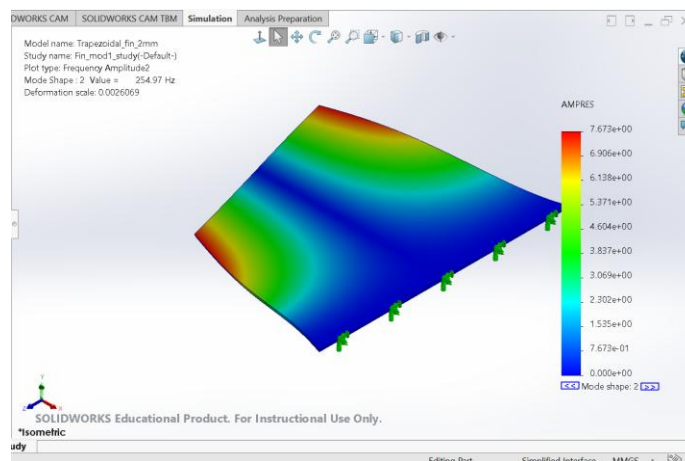


Figure 15. Von Mises stress distribution for the 100 m/s reference load case, with a maximum stress of 31.50 MPa near the fixed root.

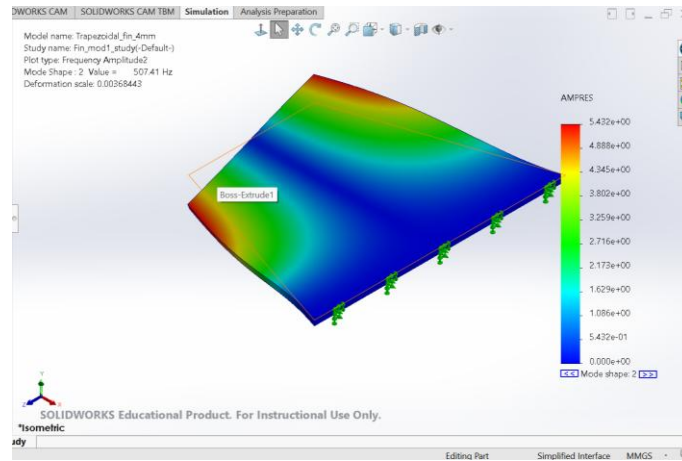


**Figure 16. Resultant displacement distribution for the 100 m/s reference load case, with a maximum displacement of 1.716 mm near the free region.**

The maximum stress occurred near the fully fixed root, while the maximum displacement occurred toward the free region. The numerical displacement corresponds to approximately 1.14% of the 150 mm span. This case is retained as a conservative unit-coefficient reference for design comparisons; it is not treated as a representative pressure distribution for a specific flight angle of attack.

### 7.3 Coefficient-Based Finite-Wing Screening

To improve the physical basis of the aerodynamic load, the baseline trapezoidal fin was treated as a finite low-aspect-ratio lifting surface. The planform area is  $S = 0.0225 \text{ m}^2$  and the aspect ratio is  $AR = b^2/S = 1.00$ . A two-dimensional thin-airfoil slope  $a_0 = 2\pi \text{ rad}^{-1}$  was corrected using a first-order finite-wing relation with Oswald efficiency  $e = 0.80$ , giving  $a \approx 1.795 \text{ rad}^{-1}$ . The small-angle coefficient estimate was then  $C_L = a\alpha$ .

$$a = a_0/[1 + a_0/(\pi e AR)] \approx 1.795 \text{ rad}^{-1}$$

$$p_e q = C_L q, q = \frac{1}{2} \rho V^2$$

At 100 m/s,  $q = 6125 \text{ Pa}$ . Equivalent pressures were calculated for  $2^\circ$ ,  $5^\circ$ , and  $10^\circ$  angle of attack. The  $5^\circ$  case was rerun in SOLIDWORKS at 960 Pa to verify the coefficient-based load and the expected linear structural scaling.

| Angle of attack | C <sub>L</sub> | Equivalent pressure | Max stress | Max displacement | Static FOS    |
|-----------------|----------------|---------------------|------------|------------------|---------------|
| $2^\circ$       | 0.0627         | 384 Pa              | 1.97 MPa   | 0.108 mm         | $\approx 140$ |
| $5^\circ$       | 0.1567         | 960 Pa              | 4.94 MPa   | 0.269 mm         | 55.9          |
| $10^\circ$      | 0.3133         | 1919 Pa             | 9.87 MPa   | 0.538 mm         | 28.0          |

The  $5^\circ$  finite-element solution produced 4.938 MPa maximum stress and 0.269 mm maximum displacement, matching the linear predictions of approximately 4.94 MPa and 0.269 mm essentially exactly. Because the static model is linear, the  $2^\circ$  and  $10^\circ$  values were then obtained by validated pressure scaling rather than redundant finite-element reruns.

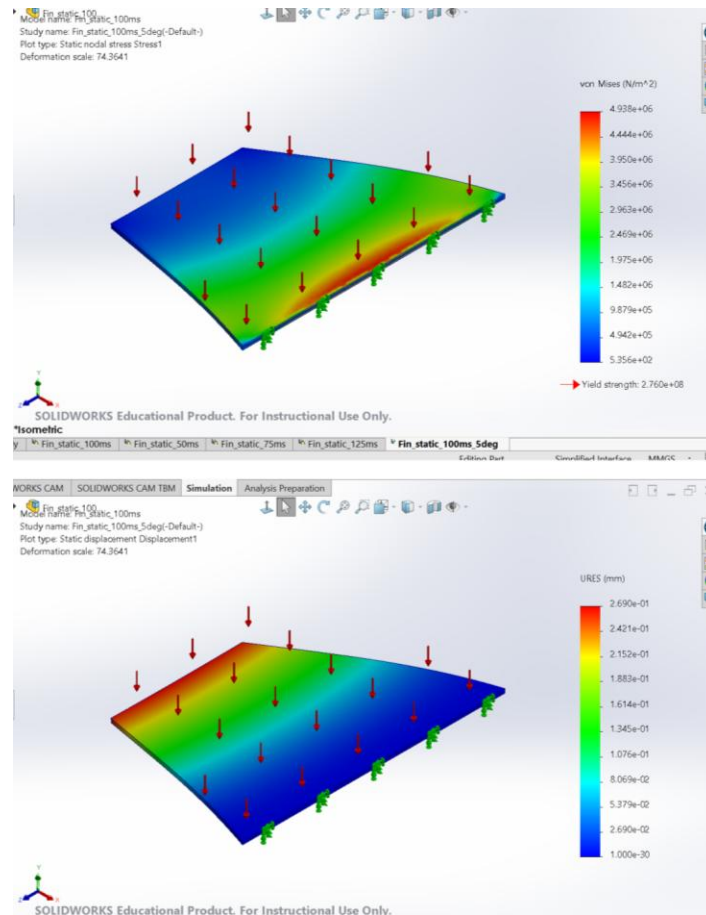


**Figure 16A. SOLIDWORKS validation of the 5° angle-of-attack coefficient-based load case at 100 m/s: 960 Pa equivalent pressure, 4.938 MPa maximum von Mises stress, and 0.269 mm maximum displacement.**

The coefficient-based cases are still first-order screening loads. They use a finite-wing correction and a uniform equivalent pressure rather than a detailed chordwise/spanwise pressure distribution, and they do not include body-fin interference, compressibility, or transonic effects. Nevertheless, they provide a substantially stronger physical basis than interpreting  $p = q$  as the actual flight pressure.

## 7.4 Static Mesh Sensitivity

| Result               | Original mesh | Refined mesh | Change |
|----------------------|---------------|--------------|--------|
| Maximum stress       | 30.92 MPa     | 31.50 MPa    | +1.88% |
| Maximum displacement | 1.713 mm      | 1.716 mm     | +0.18% |

The displacement result was essentially unchanged and the maximum stress changed by less than 2%. The peak root stress should still be interpreted cautiously because a perfectly fixed boundary can create a localized stress concentration, but the global static response was sufficiently stable for comparative use.

## 7.5 Unit-Coefficient Flight-Speed Sensitivity

| Speed  | Reference pressure $p=q$ (Pa) | Maximum stress | Maximum displacement | Approx. FOS |
|--------|-------------------------------|----------------|----------------------|-------------|
| 50 m/s | 1531.25 Pa                    | 7.876 MPa      | 0.4290 mm            | 35.04       |
| 75 m/s | 3445.31 Pa                    | 17.72 MPa      | 0.9653 mm            | 15.58       |

|         |            |           |          |      |
|---------|------------|-----------|----------|------|
| 100 m/s | 6125.00 Pa | 31.50 MPa | 1.716 mm | 8.76 |
| 125 m/s | 9570.31 Pa | 49.22 MPa | 2.681 mm | 5.61 |

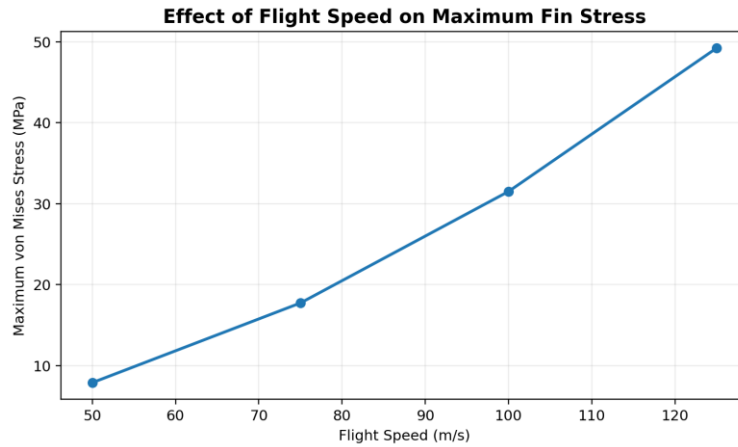


Figure 17. Maximum von Mises stress versus flight speed.

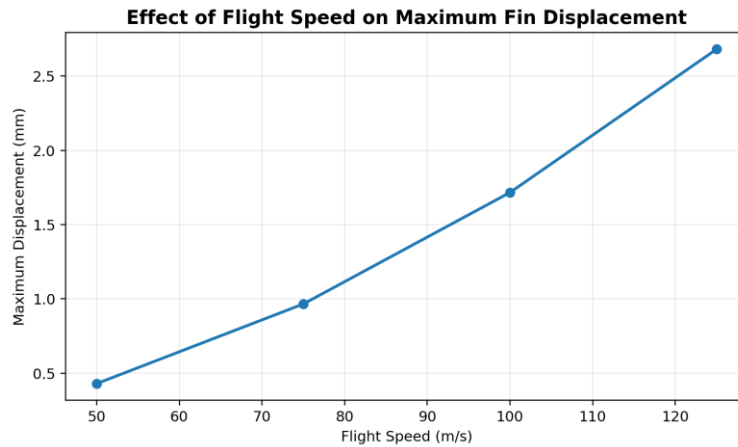


Figure 18. Maximum displacement versus flight speed.

$$\sigma \propto q \propto V^2 \quad \text{and} \quad u \propto q \propto V^2$$

## 7.6 Phase 5 Engineering Conclusions

- The 100 m/s unit-coefficient reference case produced 31.50 MPa maximum stress and 1.716 mm maximum displacement.
- The refined mesh changed the static results only slightly.
- Stress and displacement followed the expected  $V^2$  trend across the unit-coefficient speed sweep.
- 
- The coefficient-based 5° case reduced the equivalent pressure to 960 Pa and was validated in SOLIDWORKS at 4.94 MPa stress and 0.269 mm displacement; both load models remain screening approximations rather than CFD-derived flight loads.

## 8. Phase 6 — Thickness Structural Trade Study

### 8.1 Objective

Phase 6 combined modal and static results to determine how thickness affects mass, vibration characteristics, stress, and displacement under the same 100 m/s unit-coefficient reference pressure of 6125 Pa. The reference pressure is intentionally conservative and is used for internally consistent thickness comparison; Section 7.3 provides the more physically motivated coefficient-based flight-load screening.

### 8.2 Complete 2–3–4 mm Comparison

| Thickness | Mass     | Mode 1    | Mode 2    | Max stress | Max displacement | Approx. FOS |
|-----------|----------|-----------|-----------|------------|------------------|-------------|
| 2 mm      | 121.50 g | 90.83 Hz  | 254.97 Hz | 70.87 MPa  | 5.788 mm         | 3.90        |
| 3 mm      | 182.25 g | 136.19 Hz | 381.54 Hz | 31.50 MPa  | 1.716 mm         | 8.76        |
| 4 mm      | 243.00 g | 181.51 Hz | 507.41 Hz | 17.73 MPa  | 0.7253 mm        | 15.57       |

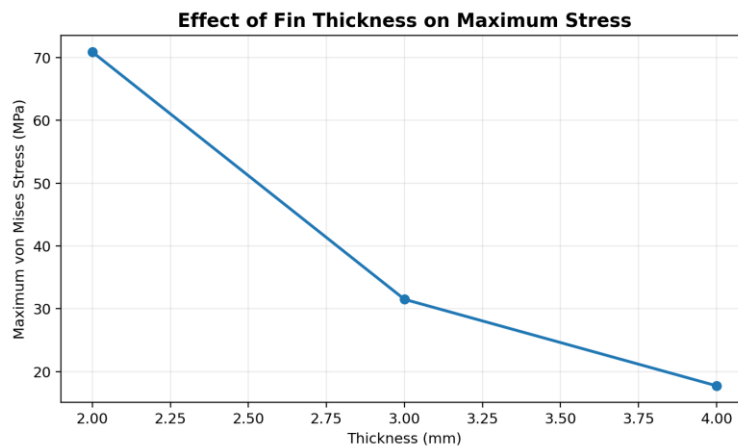


Figure 19. Effect of fin thickness on maximum stress at 100 m/s.

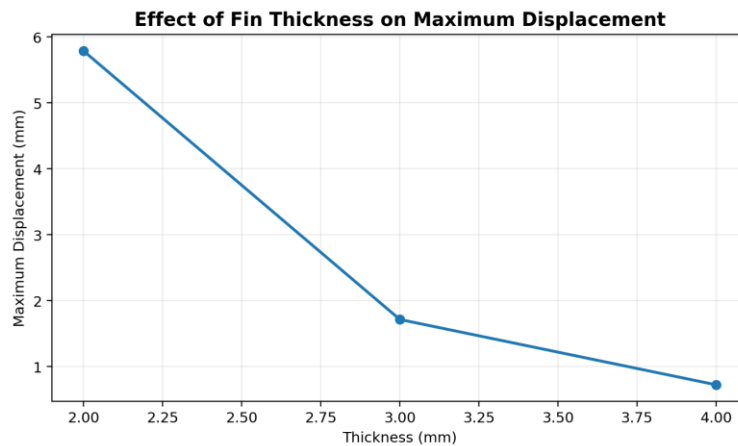


Figure 20. Effect of fin thickness on maximum displacement at 100 m/s.

Increasing thickness from 2 to 3 mm reduced maximum stress by approximately 55.6% and maximum displacement by approximately 70.4%. Increasing thickness from 3 to 4 mm reduced stress to 17.73 MPa and displacement to 0.7253 mm, while increasing mass by 33.3%.

*Stress reduction, 2→3 mm =  $(70.87 - 31.50)/70.87 \times 100 \approx 55.6\%$*

*Displacement reduction, 2→3 mm =  $(5.788 - 1.716)/5.788 \times 100 \approx 70.4\%$*

### 8.3 Phase 6 Engineering Conclusions

- 
- The 4 mm fin provides the highest stiffness, highest modal frequencies, lowest stress, and lowest displacement.
- The 4 mm improvement comes with a significant mass penalty.
- The 3 mm baseline represents an intermediate engineering tradeoff rather than the strongest or lightest design.

## 9. Phase 7 — Preliminary Aeroelastic Screening

### 9.1 Objective and Scope

Phase 7 connected the structural modal results to aerodynamic flow conditions. A full flutter calculation was deliberately not claimed because classical flutter analysis requires coupled structural and unsteady-aerodynamic stiffness and damping. The present work is therefore a preliminary screening study.

### 9.2 Baseline Structural Quantities

| Quantity                             | Value     |
|--------------------------------------|-----------|
| Bending frequency $f_b$              | 136.19 Hz |
| Torsional frequency $f_t$            | 381.54 Hz |
| Torsional-to-bending ratio $f_t/f_b$ | 2.802     |

### 9.3 Thickness and Modal Separation

| Thickness | Bending frequency | Torsional frequency | $f_t/f_b$ |
|-----------|-------------------|---------------------|-----------|
| 2 mm      | 90.83 Hz          | 254.97 Hz           | 2.807     |
| 3 mm      | 136.19 Hz         | 381.54 Hz           | 2.802     |
| 4 mm      | 181.51 Hz         | 507.41 Hz           | 2.795     |

The modal-frequency ratio remained approximately constant near 2.8 while both absolute frequencies increased with thickness. Thickness therefore shifts the structural frequency scale upward without strongly changing the relative separation of the first bending and torsional modes.

### 9.4 Reduced-Frequency Analysis

$$k = \omega c / (2V) = \pi f c / V$$

The mean chord  $c = 0.150$  m was used as the characteristic chord.

| Speed   | Bending reduced frequency $k_b$ | Torsional reduced frequency $k_t$ |
|---------|---------------------------------|-----------------------------------|
| 50 m/s  | 1.284                           | 3.596                             |
| 75 m/s  | 0.856                           | 2.397                             |
| 100 m/s | 0.642                           | 1.798                             |
| 125 m/s | 0.513                           | 1.438                             |

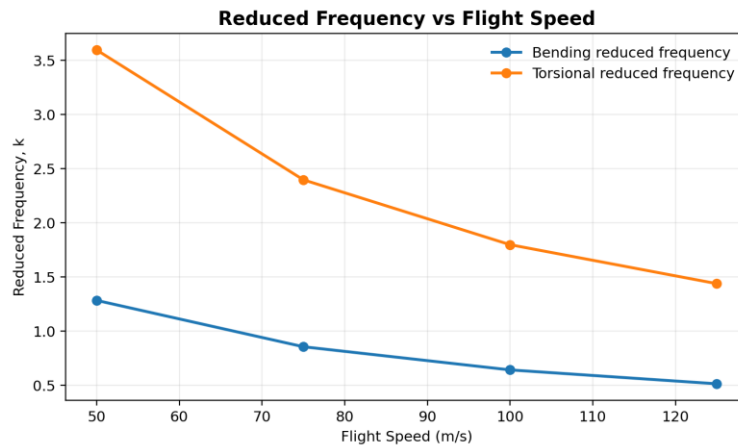


Figure 21. Reduced frequency versus flight speed for the baseline bending and torsional modes.

Reduced frequency decreased as airspeed increased because the aerodynamic flow timescale became shorter relative to the structural oscillation timescale. Reduced frequency is a useful nondimensional quantity for unsteady aerodynamics, but it does not by itself determine flutter stability.

### 9.5 Baseline Aeroelastic Screening Summary at 100 m/s

| Metric                      | Result    |
|-----------------------------|-----------|
| Bending frequency           | 136.19 Hz |
| Torsional frequency         | 381.54 Hz |
| $f_t/f_b$                   | 2.802     |
| Bending reduced frequency   | 0.642     |
| Torsional reduced frequency | 1.798     |
| Maximum static displacement | 1.716 mm  |
| Maximum static stress       | 31.50 MPa |
| Static factor of safety     | 8.76      |

### 9.6 Aeroelastic Interpretation

The preceding phases show that geometry, material, and root stiffness can substantially alter the structural frequencies of the fin. Thickness increases both bending and torsional frequencies, increasing span decreases them, and root compliance introduces much lower-frequency support-coupled modes.

These trends matter for aeroelasticity because flutter depends on structural stiffness, inertia, modal frequencies, modal coupling, structural damping, and aerodynamic stiffness/damping. The present study establishes the structural side of that problem and introduces the relevant nondimensional timescale, but it does not solve the coupled stability equations.

### 9.7 Phase 7 Conclusion

- The baseline first bending and torsional modes are separated by a frequency ratio of about 2.8.
- Reduced frequency decreases with flight speed for both modes.
- Structural design parameters substantially alter the modal frequencies relevant to aeroelastic behavior.
- No specific flutter speed is claimed from this screening analysis.
- A future flutter model should use a coupled bending–torsion formulation with structural damping and unsteady aerodynamic coefficients.

## 10. Phase 8 — Final Design Comparison and Recommendation

### 10.1 Final Thickness Comparison

| Design                | Mass     | Mode 1    | Mode 2    | Max stress @100 m/s | Max displacement @100 m/s | Approx. FOS |
|-----------------------|----------|-----------|-----------|---------------------|---------------------------|-------------|
| 2 mm 6061-T6 Aluminum | 121.50 g | 90.83 Hz  | 254.97 Hz | 70.87 MPa           | 5.788 mm                  | 3.90        |
| 3 mm 6061-T6 Aluminum | 182.25 g | 136.19 Hz | 381.54 Hz | 31.50 MPa           | 1.716 mm                  | 8.76        |
| 4 mm 6061-T6 Aluminum | 243.00 g | 181.51 Hz | 507.41 Hz | 17.73 MPa           | 0.7253 mm                 | 15.57       |

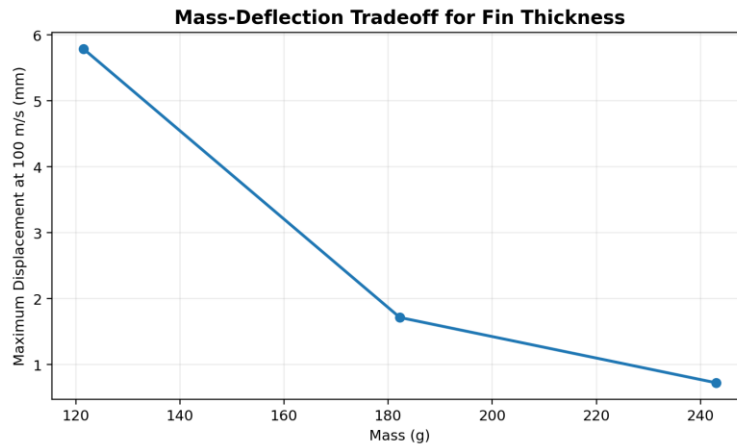


Figure 22. Mass–deflection tradeoff for the three fully evaluated aluminum thickness configurations.

### 10.2 Design Interpretation

The 4 mm fin provides the highest modal frequencies and the lowest stress and displacement. Its drawback is mass: 243.00 g compared with 182.25 g for the 3 mm baseline, an increase of approximately 33.3%.

### 10.3 Overall Project Findings

- Thickness is a strong structural lever but does not materially improve modal frequency per unit mass for solid aluminum.
- Span is one of the strongest geometric drivers of flexibility.
- The corrected constant-area sweep study showed that isolated tip-chord offset changed the first two natural frequencies by less than 1% over the tested range.
- Carbon/epoxy showed the best simplified stiffness-to-mass performance, although the composite model requires major refinement before design use.
- Root attachment compliance can dominate the lowest system frequencies; a first-order adhesive estimate of  $1.0 \times 10^{10}$  N/m produced 128.64 Hz, within about 5.5% of the fully fixed reference.
- Static stress and displacement scale approximately with  $V^2$  under the simplified dynamic-pressure model.
- The 3 mm aluminum fin provides the most balanced fully evaluated thickness configuration.
- The project establishes a credible structural foundation for a later coupled aeroelastic/flutter analysis.

## 11. Overall Limitations

- The aerodynamic loading remains an equivalent uniform-pressure approximation. A finite-wing coefficient model was added to tie load magnitude to angle of attack, but the study still does not contain a CFD-derived chordwise/spanwise pressure distribution or body-fin interference effects.
- The baseline fully fixed root is an idealization.
- The elastic-support study began as a parametric sweep and was later supplemented by a first-order adhesive shear-stiffness estimate. The estimate is still not a substitute for a detailed fin-tab/adhesive/airframe joint model or experimental calibration.
- No explicit fin tab, fillet, adhesive layer, airframe shell, or attachment hardware was modeled.
- The composite cases used simplified isotropic equivalent properties rather than orthotropic laminate definitions.
- Structural damping was not measured or included in the modal screening.
- Aerodynamic damping and unsteady aerodynamic stiffness were not calculated.
- Mode tracking across flexible-root cases was based on qualitative deformation behavior rather than a formal Modal Assurance Criterion.
- The static factor-of-safety calculations use a 6061-T6 yield strength of 276 MPa. Actual allowable strength depends on material certification, temper, manufacturing condition, and design safety requirements.
- No formal coupled flutter eigenvalue analysis was performed.

### 11.1 Fatigue and Cyclic-Loading Considerations

The present structural assessment is based primarily on linear static loading and modal characteristics and therefore does not directly predict fatigue life. Rocket fins may experience cyclic loading from propulsion-induced vibration, aerodynamic excitation, transient maneuvers, and repeated flight events. The corrected 6061-T6 static factors of safety are high for the investigated cases, but static yield margin alone does not guarantee adequate fatigue performance.

A quantitative fatigue-life assessment would require a representative stress-time history, expected cycle count, 6061-T6 S-N data, an appropriate mean-stress correction, and local stress concentrations associated with the attachment. Those inputs were not available in the present study, so fatigue is treated as a future validation requirement rather than estimated from unsupported assumptions.

The modal results remain relevant because they identify frequencies at which dynamic amplification could occur. Experimental modal testing, vibration measurements, or flight-load data could later be used to determine whether excitation occurs near the first bending or torsional modes and to establish the cyclic stress ranges required for fatigue analysis.

## 12. Recommended Future Work

- Create a detailed fin-tab, fillet, adhesive, and airframe attachment model.
- Refine the first-order attachment-stiffness estimate using explicit fin-tab, adhesive, fillet, and airframe-shell geometry, then calibrate the model experimentally if possible.
- Perform CFD to replace the equivalent uniform-pressure model with resolved pressure distributions over relevant Mach numbers and angles of attack, including body-fin interference where practical.
- Use orthotropic lamina properties and a defined stacking sequence for composite candidates.

- Perform experimental modal testing to validate natural frequencies, mode shapes, and structural damping.
- Use a Modal Assurance Criterion to track corresponding modes across design variations.
- If a true flutter boundary becomes a project objective, implement a two-degree-of-freedom bending-torsion aeroelastic model with structural damping and unsteady aerodynamic coefficients.
- Extend the aeroelastic model to include unsteady aerodynamic stiffness and damping and solve for damping versus flight speed.
- Compare the predicted critical conditions with the expected vehicle flight envelope.
- Where possible, validate static and dynamic loads using wind-tunnel or flight-test data.

## 13. Final Conclusion

The project developed a progressive structural and preliminary aeroelastic investigation of a rocket fin using SOLIDWORKS Simulation, analytical validation, and Excel-based engineering analysis. The baseline finite-element model was validated through both an analytical first-bending estimate and mesh-refinement study before being used for parameter sweeps.

The results demonstrated that thickness, span, material, and root attachment stiffness materially influence structural response. Thickness increased modal frequencies and reduced static stress and displacement but added mass proportionally. Span strongly reduced natural frequencies as it increased. The corrected constant-area sweep study showed that isolated sweep changed the first two modal frequencies by less than 1% over the tested range. The simplified carbon/epoxy equivalent produced the strongest screening stiffness-to-mass response, while the root-stiffness study showed that attachment compliance can strongly alter low-frequency behavior and that a first-order  $1.0 \times 10^{10}$  N/m adhesive estimate approaches the fully fixed modal limit.

The aeroelastic screening phase connected modal frequencies to flight speed through reduced frequency and reinforced the need to distinguish structural modal analysis from a true flutter solution. The report intentionally makes no unsupported flutter-speed claim. Instead, it provides a defensible structural foundation for future coupled aeroelastic modeling, CFD-derived loading, detailed attachment analysis, and experimental validation.

## 14. References

- [1] D. J. Inman, Engineering Vibration, 4th ed. Pearson, 2014.
- [2] I. H. Abbott and A. E. von Doenhoff, Theory of Wing Sections: Including a Summary of Airfoil Data. New York, NY, USA: Dover Publications, 1959.
- [3] R. Perry III, “Theodorsen’s and Garrick’s Flutter Calculations: Revisited,” NASA Technical Reports Server, 2024.
- [4] S. J. Weiss, K. Tseng, and L. Morino, “State-Space Formulations for Flutter Analysis,” AIAA Paper 77-117, 1977.
- [5] MatWeb, “Goodfellow E-Glass/Epoxy Composite,” material property data.
- [6] MatWeb, “Goodfellow Carbon/Epoxy Composite Tube,” material property data.

## Appendix A — Governing Equations and Assumptions

### A.1 Analytical Modal Check

$$A = bt$$

$$I = bt^3/12$$

$$f_1 = [1.875^2/(2\pi L^2)]\sqrt{(EI/\rho A)}$$

### A.2 Aerodynamic Load

$$q = \frac{1}{2}\rho V^2$$

### A.3 Static Factor of Safety

$$FOS = \sigma_y/\sigma_{max}$$

### A.4 Reduced Frequency

$$k = \omega c/(2V) = \pi f c/V$$

### A.5 Principal Assumptions

| Assumption           | Treatment                                                                                       |
|----------------------|-------------------------------------------------------------------------------------------------|
| Air density          | 1.225 kg/m <sup>3</sup>                                                                         |
| Characteristic chord | 0.150 m mean chord                                                                              |
| Baseline root        | Fully fixed                                                                                     |
| Elastic support      | Equal total normal and shear stiffness                                                          |
| Static load          | Unit-coefficient p=q reference plus coefficient-based finite-wing equivalent pressure screening |
| Composite properties | Simplified isotropic equivalents                                                                |
| Flutter assessment   | Preliminary screening only; no flutter speed claimed                                            |