

Aircraft Longitudinal Pitch Control System

Modeling, Classical and State-Space Control Design, Actuator-Constrained Simulation, and Robustness Analysis

Final Engineering Project Report

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Abstract

This project develops and evaluates a longitudinal aircraft pitch-control system using a documented Boeing 747 linear small-perturbation model at a specified cruise operating condition. The study progresses from open-loop flight-dynamics characterization through classical feedback design, MATLAB–Simulink cross-validation, actuator-constrained simulation, robustness assessment, and full-state optimal control. The reconstructed bare-airframe model exhibited a stable short-period mode with a natural frequency of 0.9617 rad/s and damping ratio of 0.3865, together with a lightly damped phugoid mode with a natural frequency of 0.0673 rad/s and damping ratio of 0.0489.

Initial elevator-only simulations revealed that long-duration pitch maneuvers produced excessive forward-speed departure from the trim condition. The original two-input aircraft formulation was therefore restored using the documented thrust-control channel, with pitch-dependent thrust feedforward and forward-speed feedback used to maintain the validity of the small-perturbation model. For the nominal 5° pitch command, the resulting maximum forward-speed perturbation remained below 0.23% of the 235.9 m/s trim speed.

A filtered PID controller was retained as the classical benchmark. Direct actuator-demand analysis showed that an unshaped 5° command requested approximately 185°/s of elevator motion despite the adopted 37°/s actuator-rate limit. A first-order command shaper with a 0.10 s time constant reduced the nominal requested rate to 31.57°/s while preserving satisfactory response. The improved PID achieved 15.33% overshoot and a 14.07 s settling time nominally, and 16.23% overshoot with a 15.09 s settling time under 20% reduced elevator effectiveness.

A tuned full-state LQI controller was evaluated under the same corrected aircraft, thrust-compensation, and actuator assumptions. For the raw 5° command, it achieved 0.682% overshoot, a 5.58 s settling time, and a peak requested elevator rate of 5.32°/s. Under 20% reduced elevator effectiveness, settling time remained 5.49 s with 1.65% overshoot. Frequency-domain analysis produced a 62.31° phase margin and 32.24 dB gain margin for the PID, while the LQI actuator-channel return ratio had a 66.62° phase margin and no finite gain-margin crossover.

Additional validation included 2° and 5° step commands, a 5° two-second ramp, longitudinal internal-state assessment, and 500 reproducible $\pm 10\%$ aerodynamic-model uncertainty trials per controller. All LQI trials satisfied the defined project criteria, while 475 of 500 PID trials satisfied all criteria; all PID trials nevertheless remained stable and tracked the commanded pitch. The final results show that both controllers provide feasible longitudinal pitch control within the modeled operating region, while the LQI provides substantially faster transient response, lower actuator demand, and greater robustness at the cost of requiring full-state feedback.

Table of Contents

Abstract	i
List of Figures	vii
List of Tables.....	viii
1. Introduction and Objectives.....	1
1.1 Background and Motivation	1
1.2 Project Objective	1
1.3 Aircraft Modeling Scope	1
1.4 Control-System Development Objectives.....	2
1.5 Performance and Engineering Evaluation Criteria	3
1.6 Computational Approach	4
1.7 Project Outcome Sought	4
2. Aircraft Model and Assumptions	5
2.1 Baseline Aircraft Model	5
2.2 Longitudinal State Definition.....	5
2.3 Elevator Sign Convention.....	6
2.4 State-Space Representation.....	6
2.5 Model Reconstruction	6
2.6 Numerical Validation of the Aircraft Model	7
2.7 Frozen Baseline Model	7
2.8 Final Thrust and Airspeed Augmentation	7
2.9 Modeling Assumptions and Limitations.....	8
3. Open-Loop Longitudinal Dynamics	9
3.1 Objective	9
3.2 Open-Loop Poles and Stability.....	9
3.3 Longitudinal Mode Identification.....	9
3.4 Elevator-to-Pitch Open-Loop Test	10
3.5 Open-Loop Time-Domain Response	10
3.6 Modal Decay and Mathematical Settling Estimate	11
3.7 Forward-Speed Validity Check.....	11
3.8 Open-Loop Validation Summary	11
3.9 Engineering Interpretation	12
4. Classical Pitch-Control Design.....	12
4.1 Control Objective.....	12

4.2 Feedback Sign Convention	13
4.3 Proportional Control	13
4.4 Proportional-Integral Control.....	13
4.5 Filtered PID Architecture	14
4.6 Ideal-Actuator PID Performance	14
4.7 P, PI, and PID Comparison.....	14
4.8 Quantitative Improvement of PID	15
4.9 Initial Project Performance Screening	16
4.10 Classical Controller Selection	16
5. MATLAB/Simulink Model Validation	17
5.1 Validation Objective.....	17
5.2 Baseline Aircraft Simulink Model.....	17
5.3 Numerical Comparison Method.....	17
5.4 Baseline Aircraft Validation Results	18
5.5 PID Implementation Validation	18
5.6 LQI Architecture Validation.....	19
5.7 Reproducibility	19
5.8 Validation Outcome	20
6. Actuator Dynamics and Physical Constraints.....	21
6.1 Motivation	21
6.2 Elevator Actuator Model	21
6.3 Effect of First-Order Actuator Lag.....	21
6.4 Initial Actuator Feasibility Audit	22
6.5 Nonlinear Actuator Model.....	22
6.6 Original PID Under Physical Constraints.....	22
6.7 Final PID Architecture	22
6.8 Reference-Command Shaping.....	23
6.9 Final Reference-Shaped PID Performance.....	24
6.10 Reduced-Elevator-Effectiveness Check	25
6.11 Engineering Interpretation	25
7. Disturbance Rejection	26
7.1 Objective.....	26
7.2 Preliminary Unconstrained Disturbance Study	26
7.3 Preliminary Disturbance Results.....	26

7.4 Actuator-Constrained Disturbance Model.....	27
7.5 Constrained +1 ° Disturbance.....	27
7.6 Constrained +2 ° Disturbance.....	27
7.7 Actuator Demand During Disturbance Rejection.....	28
7.8 Preliminary and Constrained Comparison.....	28
7.9 Engineering Interpretation	29
8. Robustness and Uncertainty Analysis.....	29
8.1 Objective.....	29
8.2 Final Uncertainty Model	30
8.3 Interpretation of the Uncertainty Distribution	30
8.4 Robustness Evaluation Criteria.....	30
8.5 PID Uncertainty Results.....	31
8.6 LQI Uncertainty Results.....	32
8.7 Summary of Numerical Uncertainty Results	33
8.8 Relation to the Earlier Parameter Studies	33
8.9 Robust Stability and Performance Interpretation	34
8.10 Engineering Interpretation	34
9. Advanced State-Feedback Control.....	35
9.1 Objective.....	35
9.2 Controllability of the Longitudinal Aircraft	36
9.3 Initial LQR Design.....	36
9.4 Static Reference-Prefilter Limitation	36
9.5 LQI Formulation	36
9.6 Initial LQI Design.....	37
9.7 Long-Duration Equilibrium Validation.....	37
9.8 Initial LQI Limitations	37
9.9 Structured LQI Weight Tuning	38
9.10 Final Tuned LQI Selection	38
9.11 Nonlinear Actuator Validation.....	39
9.12 Reduced-Elevator-Effectiveness and Implementation Considerations.....	40
9.13 Engineering Interpretation	41
10. Final PID versus Tuned-LQI Comparison.....	42
10.1 Objective	42
10.2 Final Quantitative Comparison	42

10.3 Nominal Pitch-Command Tracking	43
10.4 Actuator-Rate Demand.....	45
10.5 Reduced Elevator Effectiveness	45
10.6 Internal-State Comparison	46
10.7 Numerical Uncertainty Comparison	46
10.8 Relative Performance Improvements	47
10.9 Classical Simplicity versus Advanced Performance	47
10.10 Final Controller Selection	48
11. Final System Validation.....	49
11.1 Objective	49
11.2 Frozen Final Configuration.....	49
11.3 Multi-Command Validation.....	50
11.4 Internal-State Validation.....	50
11.5 Frequency-Domain Robustness Assessment	51
11.6 Final Robustness and Engineering Assessment	52
12. Conclusions	53
12.1 Project Summary	53
12.2 Open-Loop Aircraft Dynamics.....	53
12.3 Classical Controller Development	54
12.4 Importance of Actuator Modeling and Command Shaping.....	54
12.5 Forward-Speed Validity Correction	54
12.6 Final PID Performance.....	55
12.7 Final LQI Performance	55
12.8 Robustness and Final PID–LQI Comparison.....	55
12.9 Final Conclusion	56
13. Limitations and Future Work	57
13.1 Linear Aircraft Model	57
13.2 Longitudinal Dynamics Only	57
13.3 Rigid-Aircraft Assumption.....	57
13.4 Actuator and Thrust-Augmentation Limitations.....	57
13.5 Uncertainty-Study Limitation	58
13.6 Disturbance-Model Limitation	59
13.7 Full-State Feedback Assumption	59
13.8 Sensor and Computational Effects.....	59

13.9 Gain Scheduling and Flight-Envelope Extension.....	60
13.10 Nonlinear and Six-Degree-of-Freedom Validation	60
13.11 Hardware-in-the-Loop and Experimental Validation.....	60
13.12 Recommended Future Development Path	61
References	62
Appendices	62
Appendix A — Baseline Aircraft Model	62
Appendix B — Final Controller and Actuator Configuration	63
B.1 Final Reference-Shaped PID	63
B.2 Final Tuned LQI	63
B.3 Final Elevator Actuator Model	63
B.4 Thrust/Airspeed Augmentation	64
B.5 Frozen Configuration Files	64
Appendix C — Key MATLAB and Simulink Files	64
Appendix D — Final Corrected-Architecture Validation Record	66
D.1 Final Nominal and Reduced-Effectiveness Results	67
D.2 Multi-Command Verification	67
D.4 Frequency-Domain Verification	68
D.5 Numerical Uncertainty Verification	68
D.6 Final Configuration Verification.....	68
D.7 Final Validation Status	68

List of Figures

FIGURE 1. OPEN-LOOP BOEING 747 PITCH-ANGLE RESPONSE TO A SMALL ELEVATOR PERTURBATION, SHOWING THE LIGHTLY DAMPED LONG-PERIOD PHUGOID OSCILLATION. THE RESPONSE IS USED FOR MODAL VALIDATION; LONG-DURATION PHYSICAL INTERPRETATION IS LIMITED BY FORWARD-SPEED DEPARTURE FROM THE ORIGINAL TRIM CONDITION.	10
FIGURE 2. COMPARISON OF PROPORTIONAL, PROPORTIONAL-INTEGRAL, AND FILTERED-PID PITCH RESPONSES TO THE 5° COMMAND IN THE ORIGINAL ELEVATOR-ONLY, IDEAL-ACTUATOR DEVELOPMENT MODEL. INTEGRAL ACTION ELIMINATES STEADY-STATE ERROR, WHILE DERIVATIVE FEEDBACK SUBSTANTIALLY REDUCES OVERSHOOT AND SETTLING TIME.	15
FIGURE 3. MATLAB AND SIMULINK OPEN-LOOP BOEING 747 PITCH RESPONSES FOR THE SAME ELEVATOR INPUT. THE NEARLY COINCIDENT RESPONSES DEMONSTRATE NUMERICAL CONSISTENCY BETWEEN THE TWO IMPLEMENTATIONS.	18
FIGURE 4. NOMINAL 5° PITCH RESPONSE OF THE FINAL REFERENCE-SHAPED PID USING THE CORRECTED LONGITUDINAL ARCHITECTURE AND NONLINEAR ELEVATOR ACTUATOR. THE CONTROLLER SATISFIES THE ADOPTED OVERSHOOT, SETTLING-TIME, ELEVATOR-TRAVEL, AND ACTUATOR-RATE CRITERIA.	24
FIGURE 5. REQUESTED AND DELIVERED ELEVATOR MOTION FOR THE FINAL REFERENCE-SHAPED PID ARCHITECTURE. THE 0.10 S COMMAND SHAPER REDUCES THE REQUESTED ACTUATOR RATE BELOW THE ADOPTED 37°/S LIMIT, ALLOWING THE DELIVERED ELEVATOR RESPONSE TO CLOSELY FOLLOW THE CONTROLLER REQUEST WITHOUT SUSTAINED RATE SATURATION.	25
FIGURE 6. ACTUATOR-CONSTRAINED PITCH RESPONSE TO A +2° TWO-SECOND ADDITIVE CONTROL-CHANNEL DISTURBANCE. THE CLOSED-LOOP AIRCRAFT REMAINS STABLE AND RETURNS TO THE COMMANDED PITCH FOLLOWING DISTURBANCE REMOVAL.	28
FIGURE 7. NOMINAL 5° PITCH-COMMAND RESPONSE OF THE FINAL TUNED LQI USING THE CORRECTED LONGITUDINAL ARCHITECTURE AND NONLINEAR ELEVATOR ACTUATOR. THE CONTROLLER ACHIEVES RAPID TRACKING WITH APPROXIMATELY 0.68% OVERSHOOT AND 5.58 S SETTLING TIME WHILE REMAINING WELL WITHIN THE ADOPTED ACTUATOR LIMITS.	40
FIGURE 8. FINAL ACTUATOR-CONSTRAINED PID AND TUNED-LQI RESPONSES TO A 5° PITCH COMMAND. THE TUNED LQI PROVIDES SUBSTANTIALLY LOWER OVERSHOOT AND FASTER SETTLING WHILE BOTH CONTROLLERS CONVERGE TO THE COMMANDED PITCH.	44
FIGURE 9. FINAL PID AND TUNED-LQI PITCH RESPONSES WITH 20% REDUCED ELEVATOR EFFECTIVENESS. BOTH CONTROLLERS REMAIN STABLE AND SATISFY THE ADOPTED TRANSIENT CRITERIA, WHILE THE TUNED LQI RETAINS SUBSTANTIALLY LOWER OVERSHOOT AND FASTER SETTLING.	46
FIGURE 10. NOMINAL 5° COMMAND INTERNAL-STATE COMPARISON FOR THE FINAL PID AND TUNED-LQI CONTROLLERS. THE RESPONSES SHOW PITCH ANGLE TOGETHER WITH FORWARD-SPEED PERTURBATION, APPROXIMATE ANGLE-OF-ATTACK PERTURBATION, AND PITCH RATE. BOTH CONTROLLERS MAINTAIN SMALL FORWARD-SPEED DEPARTURE FROM THE TRIM CONDITION, WHILE THE TUNED LQI PRODUCES SMALLER INTERNAL-STATE EXCURSIONS.	51

List of Tables

TABLE 1. OPEN-LOOP LONGITUDINAL MODE CHARACTERISTICS..... 9

TABLE 2. P, PI, AND PID CONTROLLER PERFORMANCE COMPARISON.....14

TABLE 3. IDEAL-ELEVATOR AND FIRST-ORDER-ACTUATOR PERFORMANCE COMPARISON 21

TABLE 4. PRELIMINARY DISTURBANCE-REJECTION RESULTS.....27

TABLE 5. PRELIMINARY AND ACTUATOR-CONSTRAINED DISTURBANCE COMPARISON 28

TABLE 6. FINAL 500-TRIAL NUMERICAL UNCERTAINTY RESULTS.....33

TABLE 7. FINAL PID AND TUNED-LQI PERFORMANCE COMPARISON.....42

TABLE 8. RELATIVE PERFORMANCE IMPROVEMENTS WITH TUNED LQI 47

TABLE 9. FINAL MULTI-COMMAND VALIDATION MATRIX.....50

TABLE 10. FINAL CORRECTED-ARCHITECTURE VALIDATION SUMMARY 67

1. Introduction and Objectives

1.1 Background and Motivation

Aircraft longitudinal control governs motion in the vertical plane and is particularly concerned with the relationship between elevator deflection, pitch rate, and aircraft pitch attitude. A pitch-control system must provide accurate command tracking while maintaining stable aircraft motion and avoiding excessive control-surface activity.

Although an aircraft may be naturally stable at a particular flight condition, open-loop stability alone does not guarantee satisfactory command-following performance. Lightly damped longitudinal modes can produce long settling times and significant pitch excursions following control inputs or external disturbances. Feedback control is therefore used to improve transient response, tracking accuracy, disturbance rejection, and overall dynamic behavior.

This project investigates longitudinal pitch control using a documented Boeing 747 small-perturbation model. The analysis is performed using MATLAB and Simulink and progresses from open-loop aircraft characterization to classical feedback control, actuator-constrained simulation, robustness analysis, and advanced full-state feedback control.

1.2 Project Objective

The primary objective of this project is to develop, simulate, and evaluate a longitudinal aircraft pitch-control system using a documented Boeing 747 linear small-perturbation model.

Pitch angle, θ , is the principal controlled variable. Elevator deflection, δ_e , provides the primary pitch-control action, while the documented thrust-control channel is retained in the final architecture to limit forward-speed departure from the selected trim condition during commanded pitch maneuvers.

The project evaluates not only whether a controller can track a commanded pitch angle, but also whether the resulting aircraft response is dynamically stable, compatible with the assumptions of the linearized model, feasible within the adopted elevator-actuator limits, and reasonably robust to modeled uncertainty.

Two final controller architectures are compared:

- a filtered PID controller with reference shaping;
- a full-state Linear Quadratic Integral (LQI) controller.

The comparison considers tracking performance, actuator demand, longitudinal-state behavior, frequency-domain robustness, reduced elevator effectiveness, and numerical aerodynamic-model uncertainty.

1.3 Aircraft Modeling Scope

The aircraft is represented using a linear longitudinal small-perturbation model about a specified trimmed cruise condition.

The longitudinal state vector is $x = [u \quad w \quad q \quad \theta]^T$, where:

- u is the forward-velocity perturbation;

- w is the vertical body-axis velocity perturbation;
- q is the pitch-rate perturbation;
- θ is the pitch-angle perturbation.

The documented aircraft formulation contains both elevator and thrust perturbation inputs. Elevator deflection is used as the primary feedback-control input for pitch regulation. During the final validation stages, the thrust channel is restored through a supplementary airspeed-hold augmentation consisting of pitch-dependent feedforward and forward-speed feedback.

This augmentation is introduced because elevator-only long-duration pitch maneuvers can produce forward-speed departures large enough to weaken the validity of the linear small-perturbation assumption. The corrected architecture therefore retains the aircraft near its baseline forward-speed trim while the elevator controller regulates pitch.

The model represents rigid-body longitudinal motion near one specified cruise operating point. Lateral-directional dynamics, structural flexibility, aeroelastic behavior, stall, large-angle nonlinear aerodynamics, propulsion-system dynamics, and complete aircraft flight-envelope behavior remain outside the scope of the present study.

Accordingly, the results should be interpreted as a comparative flight-control analysis near the selected trim condition rather than as a complete high-fidelity Boeing 747 simulation.

1.4 Control-System Development Objectives

The controller-development process was structured so that model fidelity and control-system sophistication increased progressively.

The study includes:

1. reconstruction and validation of the documented longitudinal aircraft model;
2. characterization of the natural short-period and phugoid modes;
3. development and comparison of proportional, proportional-integral, and filtered proportional-integral-derivative control;
4. MATLAB and Simulink cross-validation;
5. introduction of first-order elevator-actuator dynamics, travel limits, and rate limits;
6. direct comparison of requested and physically delivered actuator motion;
7. development of reference shaping to avoid excessive commanded actuator rate;
8. restoration of the thrust-control channel to maintain forward-speed validity;
9. disturbance-rejection assessment;
10. development of LQR and LQI full-state feedback control;
11. controlled PID-versus-LQI comparison under matched aircraft and actuator assumptions;

12. classical PID and LQI actuator-channel frequency-domain robustness analysis;
13. validation using multiple pitch-command profiles and internal longitudinal states;
14. numerical model-uncertainty assessment using repeated Monte Carlo simulations.

This progression allows each design decision to be evaluated against an established baseline and demonstrates how conclusions can change when additional physical and implementation constraints are introduced.

1.5 Performance and Engineering Evaluation Criteria

Controller performance is evaluated quantitatively rather than solely through visual inspection of pitch-response plots.

The principal evaluation quantities are:

- steady-state pitch tracking error;
- percentage overshoot;
- peak pitch response;
- 2% settling time;
- elevator deflection;
- requested elevator-deflection rate;
- delivered elevator-deflection rate;
- closed-loop stability;
- forward-speed perturbation;
- vertical-velocity and approximate angle-of-attack perturbations;
- pitch-rate response;
- sensitivity to reduced elevator effectiveness;
- performance under aerodynamic-model uncertainty;
- gain margin and phase margin where applicable;
- closed-loop bandwidth;
- peak sensitivity.

For the principal pitch-command studies, project-level transient screening targets of $M_p \leq 20\%$ and $t_s \leq 30$ s are retained.

The adopted elevator actuator has travel limits of $-23^\circ \leq \delta_e \leq +17^\circ$ and a maximum deflection-rate magnitude of $|\dot{\delta}_e| \leq 37^\circ/\text{s}$.

These values are engineering constraints adopted for this project and are not presented as certification requirements for the specific aircraft model.

Because the aircraft model is linearized about a fixed trim condition, forward-speed and internal-state excursions are also monitored to ensure that the simulated maneuver remains reasonably close to the operating point from which the model was derived.

1.6 Computational Approach

MATLAB is used for:

- state-space implementation;
- eigenvalue and modal analysis;
- controller design;
- time-domain simulation;
- numerical parameter sweeps;
- reference-shaper selection;
- actuator-demand analysis;
- frequency-domain robustness calculations;
- Monte Carlo uncertainty simulation;
- quantitative performance evaluation.

Simulink is used to construct explicit closed-loop control architectures and to represent actuator dynamics, nonlinear travel and rate limits, disturbances, and controller signal paths.

Independent MATLAB–Simulink comparisons are performed at several stages to verify that differences in system response arise from intended modeling or controller changes rather than inconsistent software implementations.

Validated aircraft and controller configurations are stored separately from analysis scripts. The baseline aircraft is frozen in `B747_longitudinal_baseline.mat`, while final controller and actuator parameters are stored in `final_control_configuration.mat`. This separation improves reproducibility and prevents later analysis scripts from silently modifying previously validated parameters.

1.7 Project Outcome Sought

The purpose of the project is not simply to minimize pitch settling time.

The final control system must balance:

tracking performance + stability + actuator feasibility + model validity + robustness + implementation complexity

The final controllers are therefore evaluated under nominal operation, actuator constraints, reduced elevator effectiveness, multiple command profiles, longitudinal-state monitoring, and numerical model uncertainty.

The final controller selection is based on the complete engineering evidence rather than on a single nominal pitch-response plot. The PID serves as the lower-complexity classical benchmark, while the LQI architecture is evaluated to determine whether the additional requirement for full-state feedback is justified by improved transient performance, reduced actuator demand, and greater robustness.

2. Aircraft Model and Assumptions

2.1 Baseline Aircraft Model

A documented Boeing 747 longitudinal dynamics model was selected as the baseline aircraft plant for the control-system study.

The model represents steady level cruise at approximately Mach 0.80 and an altitude of 40,000 ft. The corresponding trim forward speed used in the MATLAB implementation is approximately $U_0 = 235.9$ m/s.

The aircraft is represented by a linear small-perturbation model developed about this specific operating condition. Consequently, the equations describe local longitudinal dynamics near the selected trim point rather than aircraft behavior throughout the complete flight envelope.

The model was reconstructed in MATLAB from documented aircraft properties and dimensional stability and control derivatives rather than from arbitrarily selected coefficients.

The documented formulation contains two longitudinal control inputs: $u_c = \begin{bmatrix} \delta_e \\ \delta_p \end{bmatrix}$, where δ_e is elevator deflection and δ_p is the documented thrust-control perturbation variable.

Elevator deflection is retained as the primary pitch-control input. The thrust channel is used later as a supplementary longitudinal trim and forward-speed augmentation input.

2.2 Longitudinal State Definition

The longitudinal state vector is $x = [u \ w \ q \ \theta]^T$, where:

- u is the forward body-axis velocity perturbation, in m/s;
- w is the vertical body-axis velocity perturbation, in m/s;
- q is the pitch-rate perturbation, in rad/s;
- θ is the pitch-angle perturbation, in rad.

Pitch angle is the principal controlled output of the project.

The complete state vector is retained throughout the simulations even when only pitch angle is used by the classical PID controller. This allows internal longitudinal motion to be monitored and enables full-state feedback in the LQI controller.

2.3 Elevator Sign Convention

The documented aircraft model defines positive elevator deflection as elevator-down motion.

The elevator-to-pitch plant has a negative steady-state gain. Therefore, under this convention, a negative elevator deflection produces a positive steady-state pitch-angle perturbation.

Accordingly, a positive nose-up pitch command generally requires a negative physical elevator command.

This sign convention is retained consistently throughout the classical PID and full-state LQI implementations.

2.4 State-Space Representation

The longitudinal equations are written in state-space form as $\dot{x} = Ax + B_e\delta_e + B_p\delta_p$, with pitch angle selected as the primary controlled output, $y = C_\theta x$.

Using the state ordering $x = [u \quad w \quad q \quad \theta]^T$, the reconstructed baseline state matrix is approximately

$$A = \begin{bmatrix} -0.0069 & 0.0139 & 0 & -9.81 \\ -0.0905 & -0.3149 & 235.8928 & 0 \\ 0.0004 & -0.0034 & -0.4282 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The elevator-input vector is approximately $B_e = \begin{bmatrix} -5.726 \times 10^{-5} \\ -5.5079 \\ -1.1569 \\ 0 \end{bmatrix}$, while the restored thrust-control channel is

$$B_p = \begin{bmatrix} 2.943 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Pitch angle is extracted using $C_\theta = [0 \quad 0 \quad 0 \quad 1]$, $D_\theta = 0$.

The fourth state equation contains the kinematic relation $\dot{\theta} = q$.

Neither elevator nor thrust therefore changes pitch angle instantaneously. Their influence occurs through the longitudinal dynamic states.

2.5 Model Reconstruction

The state-space matrices were reconstructed in MATLAB from the documented dimensional stability and control derivatives.

The complete two-input aircraft structure was reconstructed before individual control channels were examined. This provided a traceable implementation that could be checked against:

1. the documented aircraft parameters;
2. the dimensional stability derivatives;
3. the longitudinal equations of motion;
4. the published numerical state-space representation.

The reconstructed elevator-only plant was initially used for open-loop analysis and controller development because elevator deflection was the primary pitch-control variable.

During later physical-validity testing, however, the original thrust input was restored because elevator-only sustained pitch maneuvers produced excessive forward-speed departure from the trim condition.

2.6 Numerical Validation of the Aircraft Model

The independently reconstructed state matrix was compared with the published numerical model.

Only small differences were observed, primarily because the published comparison coefficients were available at limited decimal precision. The agreement between the independently reconstructed model and the published state-space representation provided confidence that the implemented dynamics were consistent with the documented aircraft data.

Programmatic matrix-dimension checks were also used to prevent accidental modification of the model structure.

The reconstructed aircraft model was therefore accepted as the baseline plant for subsequent control-system development.

2.7 Frozen Baseline Model

Following reconstruction and validation, the aircraft model was saved as `B747_longitudinal_baseline.mat`.

The frozen file contains the validated baseline longitudinal aircraft model and the principal trim parameters used throughout the project.

The final controller parameters were stored separately in `final_control_configuration.mat`.

This separation prevents controller-development scripts from silently changing the validated aircraft model and improves reproducibility across later analyses.

The baseline file preserves the elevator-control model used during the original controller-design stages. The thrust input required for the final augmented architecture was reconstructed from the same documented longitudinal model and reintroduced explicitly during the Phase 14 validation work.

2.8 Final Thrust and Airspeed Augmentation

Long-duration elevator-only simulations revealed an important model-validity limitation.

Although the pitch response itself appeared well behaved, a sustained positive pitch perturbation generated a large negative forward-speed perturbation. For the nominal 5° maneuver, the elevator-only model produced approximately $\Delta u = -36.7$ m/s after 60 seconds, corresponding to more than 15% of the trim forward speed.

Such a departure is inconsistent with treating the aircraft as a small perturbation about the original cruise trim condition.

The documented thrust-control channel was therefore restored.

The final supplementary thrust law was $\delta_p = \delta_{p,ff} + \delta_{p,fb}$, with pitch-dependent feedforward $\delta_{p,ff} = \frac{\theta}{0.3}$

and forward-speed feedback $\delta_{p,fb} = -K_u u$, where $K_u = 0.05$.

The feedforward term follows directly from the longitudinal u -equation. The baseline state matrix contains the gravity coupling $-g\theta$, while the thrust channel contributes approximately $0.3g \delta_p$. Therefore, at constant pitch, $-g\theta + 0.3g\delta_p = 0$ gives $\delta_p = \frac{\theta}{0.3}$.

The additional forward-speed feedback suppresses transient velocity error.

For the final nominal 5° PID simulation, this augmentation reduced the maximum forward-speed perturbation to approximately 0.53 m/s, or only 0.23% of the trim speed.

The thrust variable is retained using the normalized control definition from the source model. No claim is made that the present augmentation represents a complete physical engine or autothrottle system, because propulsion-system actuator dynamics and thrust limits were not modeled.

2.9 Modeling Assumptions and Limitations

The following assumptions apply to the aircraft model and final control analysis:

- the aircraft is treated as a rigid body;
- motion is restricted to the longitudinal plane;
- the analysis is performed near a steady cruise trim condition;
- perturbations are assumed sufficiently small for linearization;
- lateral-directional dynamics are neglected;
- structural flexibility and aeroelastic effects are neglected;
- aerodynamic coefficients are treated as locally linear;
- elevator deflection is the primary pitch-control input;
- the thrust channel is used only as a supplementary forward-speed and trim augmentation;
- propulsion-system dynamics, thrust saturation, and engine-spool dynamics are not modeled;
- sensor noise, state-estimation error, and flight-control-computer delays are not explicitly represented;
- atmospheric and aircraft parameters correspond to the selected baseline operating condition.

The model therefore should not be interpreted as a complete high-fidelity Boeing 747 simulation.

The final validation explicitly monitors Δu , Δw , $\Delta \alpha$, q , θ to determine whether commanded maneuvers remain reasonably consistent with the local small-perturbation interpretation.

For the nominal 5° command, the final shaped PID produced approximately $\max |\Delta \alpha| = 3.84^\circ$, while the LQI produced approximately $\max |\Delta \alpha| = 2.78^\circ$.

These excursions are substantially more consistent with a local linear-control study than the large forward-speed departures produced by the original elevator-only long-duration simulations. Nevertheless, the results

remain valid only near the modeled operating condition and should not be extrapolated directly to large-angle maneuvers or the complete aircraft flight envelope.

3. Open-Loop Longitudinal Dynamics

3.1 Objective

Before introducing feedback control, the validated Boeing 747 longitudinal model was analyzed in open loop to establish its natural dynamic behavior.

The analysis focused on:

- open-loop stability,
- short-period and phugoid modes,
- response to a small elevator perturbation,
- steady-state elevator-to-pitch behavior,
- transient amplitude and settling,
- consistency between modal and time-domain predictions.

This open-loop characterization provides the baseline against which the closed-loop controllers are evaluated.

3.2 Open-Loop Poles and Stability

The eigenvalues of the longitudinal state matrix were calculated independently using the state matrix and the complete state-space model. Both methods produced the pole pairs $\lambda_{1,2} = -0.3717 \pm 0.8869i$ and $\lambda_{3,4} = -0.0033 \pm 0.0672i$.

All four poles have negative real parts. Therefore, the linearized aircraft is asymptotically stable at the selected cruise operating condition. However, the two pole pairs have substantially different time scales and damping characteristics.

3.3 Longitudinal Mode Identification

The faster pole pair $-0.3717 \pm 0.8869i$ was identified as the **short-period mode**. Its calculated characteristics were $\omega_n = 0.9617$ rad/s, $\zeta = 0.3865$, $T = 7.0842$ s, with an approximate exponential decay time constant of $\tau = 2.6903$ s. The slower pole pair $-0.0033 \pm 0.0672i$ was identified as the **phugoid mode**. Its characteristics were $\omega_n = 0.0673$ rad/s, $\zeta = 0.0489$, $T = 93.4971$ s, and $\tau \approx 303.03$ s.

The modal properties are summarized below.

Table 1. Open-Loop Longitudinal Mode Characteristics

Mode	Poles	ω_n rad/s	ζ	Period (s)	Decay time constant (s)
Short period	$-0.3717 \pm 0.8869i$	0.9617	0.3865	7.0842	2.6903
Phugoid	$-0.0033 \pm 0.0672i$	0.0673	0.0489	93.4971	303.03

The short-period mode decays relatively quickly, whereas the phugoid mode is only weakly damped and persists for several minutes. Thus, although the aircraft is mathematically stable, its long-period transient behavior is extremely slow.

3.4 Elevator-to-Pitch Open-Loop Test

A small -1° elevator step was applied to the bare-airframe longitudinal model to examine the qualitative elevator-to-pitch response.

Under the adopted sign convention, an elevator-up perturbation produces a positive nose-up pitch response.

The small input magnitude was selected to remain consistent with the local linearized aircraft model.

The elevator-to-pitch channel exhibits a finite low-frequency gain, and the pitch response contains both the faster short-period mode and the much slower phugoid mode identified from the aircraft eigenvalues.

The purpose of this test is therefore to connect the state-space modal analysis to the observable pitch response rather than to establish a physically valid long-duration trimmed maneuver using elevator alone.

3.5 Open-Loop Time-Domain Response

The open-loop pitch response exhibits a large, slowly decaying oscillation dominated by the lightly damped phugoid mode.

The spacing between the principal long-period response peaks is consistent with the independently calculated phugoid period of approximately 93.50 s.

This agreement provides a useful cross-check between the eigenvalue analysis and the time-domain simulation.

The response also illustrates that mathematical stability alone does not imply useful command-following performance. Even though all bare-airframe poles lie in the left-half plane, the lightly damped phugoid produces persistent long-period motion.

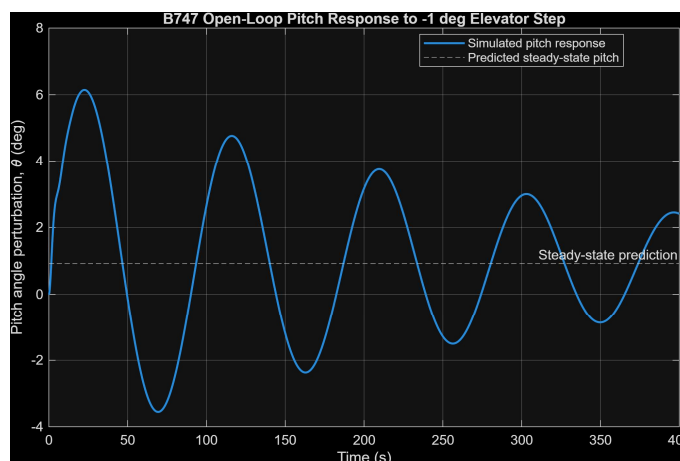


Figure 1. Open-loop Boeing 747 pitch-angle response to a small elevator perturbation, showing the lightly damped long-period phugoid oscillation. The response is used for modal validation; long-duration physical interpretation is limited by forward-speed departure from the original trim condition.

3.6 Modal Decay and Mathematical Settling Estimate

For the bare-airframe phugoid poles, $\lambda_{ph} = -0.003289 \pm 0.067202j$, the decay rate is determined by the real component $\sigma_{ph} = -0.003289 \text{ s}^{-1}$.

The corresponding exponential time constant is approximately $\tau = \frac{1}{|\sigma_{ph}|} \approx 303 \text{ s}$.

The time required for the modal amplitude to decrease by one half is $t_{1/2} = \frac{\ln 2}{|\sigma_{ph}|} \approx 210.75 \text{ s}$.

A conventional 2% exponential-envelope estimate therefore produces a settling time on the order of several time constants.

This very long value is useful as a mathematical description of the lightly damped phugoid mode. It should not, however, be interpreted as a physically validated multi-minute elevator-only flight maneuver about the original trim condition.

3.7 Forward-Speed Validity Check

A later internal-state audit revealed an important limitation of the original elevator-only open-loop simulation.

Although the pitch response continued to converge mathematically, the forward-speed perturbation did not remain small relative to the original trim condition.

For the representative 5° pitch maneuver used during later controller validation, the elevator-only aircraft produced approximately $\Delta u = -36.7 \text{ m/s}$ after 60 seconds.

Relative to the trim speed $U_0 = 235.9 \text{ m/s}$, this corresponds to approximately 15.6% departure from the baseline forward speed.

Such a deviation is too large to support a strong physical interpretation of a long-duration response using a linear model derived about the original cruise trim.

Therefore, the earlier multi-minute elevator-only settling calculation is retained only as a modal and mathematical observation. It is not used as evidence that the aircraft would physically remain near the same trim state for the complete settling interval.

3.8 Open-Loop Validation Summary

The open-loop analysis supports the following conclusions:

1. the eigenvalues calculated directly from the state matrix agree with the poles of the implemented state-space system;
2. the faster complex pair corresponds to a stable short-period mode with $\omega_{n,sp} = 0.9617 \text{ rad/s}$, $\zeta_{sp} = 0.3865$;
3. the slower complex pair corresponds to a stable but lightly damped phugoid mode with $\omega_{n,ph} = 0.0673 \text{ rad/s}$, $\zeta_{ph} = 0.0489$;
4. the observed long-period pitch oscillation is consistent with the calculated phugoid period of approximately 93.50 s;

5. the phugoid decay rate explains the mathematically slow open-loop convergence;
6. long-duration elevator-only simulation cannot be treated as a fully valid trim-preserving flight maneuver because the forward-speed perturbation becomes too large.

The modal conclusions therefore remain valid, while the physical interpretation of very long elevator-only time histories is deliberately limited.

3.9 Engineering Interpretation

The bare-airframe Boeing 747 longitudinal model is dynamically stable at the selected cruise operating condition, but its natural response is not suitable for rapid or accurate pitch-command tracking.

The short-period mode is moderately damped and decays relatively quickly, whereas the phugoid mode is lightly damped and persists over a much longer time scale.

The key engineering lesson from the open-loop study is twofold.

First, natural stability does not guarantee satisfactory control performance.

Second, a response that is mathematically valid within a linear state-space model is not automatically physically meaningful over an arbitrarily long simulation interval. Internal-state excursions must also remain sufficiently close to the trim condition on which the linearization is based.

This observation motivated two subsequent developments:

- feedback control to improve pitch-command tracking and transient response;
- restoration of the documented thrust-control channel during final validation to limit forward-speed departure and preserve the local small-perturbation interpretation.

The open-loop model is therefore used primarily for modal characterization and controller-baseline development rather than as a prediction of a sustained elevator-only maneuver over several minutes.

4. Classical Pitch-Control Design

4.1 Control Objective

This section documents the original classical-controller development using the elevator-only aircraft model and an ideal actuator; the controller is reassessed under the final corrected architecture in later sections.

After establishing the open-loop longitudinal behavior, feedback control was introduced to track a commanded aircraft pitch angle. The principal test command was $\theta_{cmd} = 5^\circ$.

Three classical controller architectures were developed sequentially: P $\rightarrow$ PI $\rightarrow$ PID

The purpose of this progression was to identify the contribution of proportional, integral, and derivative feedback before introducing actuator nonlinearities later in the study.

Performance was evaluated using:

- steady-state tracking error,

- maximum pitch angle,
- percentage overshoot,
- 2% settling time,
- closed-loop stability,
- elevator demand.

At this stage, the elevator was treated as an ideal control input. Physical actuator travel and rate limitations are introduced separately in Section 6.

4.2 Feedback Sign Convention

The pitch tracking error was defined as $e(t) = \theta_{cmd}(t) - \theta(t)$. Because the validated elevator-to-pitch plant has a negative steady-state gain, a positive nose-up pitch error requires a negative physical elevator command. For proportional feedback, the corresponding physical control direction is therefore $\delta_e = -K_p e$. This sign convention was retained throughout the P, PI, and PID designs.

4.3 Proportional Control

Proportional control was first investigated as the simplest feedback architecture. A gain sweep was used to examine the effect of K_p on tracking error, overshoot, settling behavior, and elevator demand. A representative proportional benchmark of $K_p = 2$ was selected. For the 5° pitch command, the controller produced a final pitch angle of 3.2432° , corresponding to a steady-state tracking error of 1.7568° . The maximum pitch response was 6.4128° , with approximately 28.26% command overshoot. The 2% settling time relative to the controller's own final value was approximately 189.12 s. Maximum absolute elevator demand reached 10.0° .

Proportional feedback therefore improved the aircraft response but could not provide accurate steady-state pitch tracking. This result motivated the addition of integral action.

4.4 Proportional-Integral Control

Integral feedback was introduced to eliminate the persistent tracking error.

The control law was $\delta_e = (K_p e + K_I \int e, dt)$. A two-parameter gain study was performed, and the selected PI benchmark used $K_p = 0.5$, $K_I = 0.3 \text{ s}^{-1}$. The PI controller achieved $\theta_{ss} = 5.0000^\circ$ with effectively zero steady-state error. The maximum pitch response was 6.3085° , corresponding to 26.17% overshoot. The 2% settling time decreased to approximately 67.01 s. The long-duration maximum absolute elevator command was 5.4161° .

An independent steady-state check was performed using the aircraft DC gain. Since $K_{DC} = -0.9230$, the elevator required to maintain a 5° pitch perturbation is approximately $\delta_{e,ss} \frac{5^\circ}{-0.9230} \approx -5.417^\circ$. The simulated long-duration PI elevator command converged to approximately -5.416° , providing an independent equilibrium check.

Integral action therefore solved the steady-state tracking problem, but overshoot and settling time remained relatively high.

4.5 Filtered PID Architecture

Derivative action was introduced to improve transient damping. Rather than differentiating the pitch-command step directly, the derivative term was applied to the measured pitch angle using a first-order filtered derivative, $D(s)K_D \frac{s}{T_f s + 1}$.

Applying derivative action to the measurement rather than directly to the reference reduces derivative kick at the instant of the step command. The controller architecture therefore combined:

- proportional action on pitch error,
- integral action on pitch error,
- filtered derivative feedback from measured pitch angle.

A derivative-gain sweep was followed by a targeted multi-parameter study of (K_P) , (K_I) , and (K_D) .

The initial ideal-actuator PID selected from this study used $K_P = 0.8$, $K_I = 0.5 \text{ s}^{-1}$, $K_D = 0.8 \text{ s}$, $T_f = 0.1 \text{ s}$ and was carried forward for initial controller assessment.

4.6 Ideal-Actuator PID Performance

For the 5° pitch command, the selected PID achieved $\theta_{ss} = 5.0000^\circ$ with effectively zero steady-state tracking error. The maximum pitch angle was 5.9750° , corresponding to 19.50%. The peak occurred at approximately 4.0 s , and the 2% settling time was 25.57 s. Maximum long-duration elevator magnitude was approximately 5.4162° . The closed-loop poles were -8.9334 , $-0.6556 \pm 1.0800i$, $-0.2472 \pm 0.2479i$, and -0.0111 .

All poles have negative real parts, confirming closed-loop stability for the ideal-actuator design.

4.7 P, PI, and PID Comparison

The controller-development results are summarized below.

Table 2. P, PI, and PID Controller Performance Comparison

Metric	P	PI	PID
Final pitch (deg)	3.2432	5.0000	5.0000
Steady-state error (deg)	1.7568	≈ 0	≈ 0
Maximum pitch (deg)	6.4128	6.3085	5.9750
Overshoot (%)	28.256	26.171	19.499
2% settling time (s)	189.12	67.009	25.57
Maximum elevator magnitude (deg)	10.0000	5.4161	5.4162

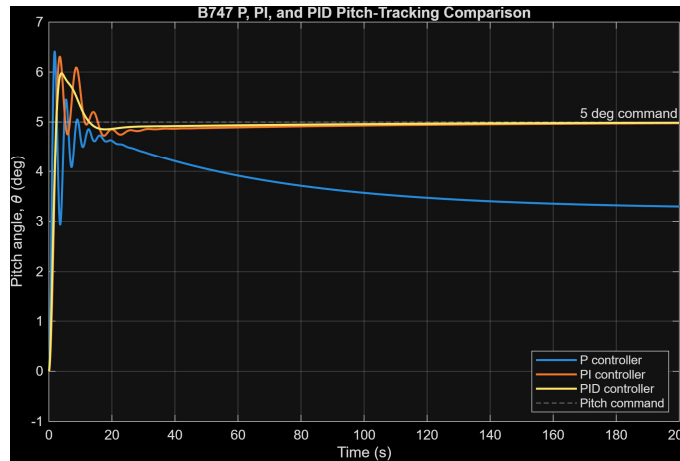


Figure 2. Comparison of proportional, proportional-integral, and filtered-PID pitch responses to the 5° command in the original elevator-only, ideal-actuator development model. Integral action eliminates steady-state error, while derivative feedback substantially reduces overshoot and settling time.

The progression demonstrates the distinct contribution of each control term. Proportional feedback provides immediate correction but retains substantial steady-state error. Integral feedback eliminates this error by generating the persistent elevator bias required to maintain the commanded pitch.

Derivative feedback improves damping and allows substantially faster transient behavior while preserving zero steady-state error.

4.8 Quantitative Improvement of PID

Within the original elevator-only, ideal-actuator development model, the filtered PID controller provided the strongest classical response among the P, PI, and PID architectures investigated.

Relative to the selected PI controller, PID reduced overshoot and substantially reduced settling time while maintaining effectively zero steady-state pitch error.

Relative to the proportional benchmark, PID also provided considerably faster convergence while avoiding the persistent tracking error associated with proportional control alone.

These comparisons established the filtered PID architecture as the preferred classical controller structure for the next stage of the project.

However, the numerical results in this section should be interpreted as controller-development results rather than final aircraft-system performance. At this stage:

- the elevator was treated as an ideal control input;
- actuator travel and rate limits were not yet enforced;
- the thrust-control channel was not active;
- internal forward-speed validity had not yet been audited;
- reference shaping had not yet been introduced.

The controller was therefore carried forward as a classical baseline subject to later physical and model-validity reassessment.

4.9 Initial Project Performance Screening

Two project-level transient targets were used during the early controller-development phase: $M_p \leq 20\%$ and $t_s \leq 30$ s.

Under the original ideal-actuator, elevator-only architecture, the selected PID satisfied both transient screening targets while maintaining effectively zero steady-state pitch error.

This result justified advancing the PID architecture to actuator-constrained simulation.

The criteria are retained throughout the project as consistent engineering comparison thresholds. They are not aircraft certification requirements and do not, by themselves, establish physical validity of the complete aircraft response.

Later sections therefore supplement these criteria with actuator-demand, internal-state, forward-speed, robustness, and frequency-domain assessments.

4.10 Classical Controller Selection

The proportional controller was rejected as the principal classical design because it retained significant steady-state tracking error.

The PI controller eliminated steady-state error but retained comparatively high overshoot and slower transient response.

The filtered PID controller provided the strongest overall combination of tracking accuracy, transient response, and control effort within the initial classical-controller study.

It was therefore selected as the classical architecture for subsequent physical validation.

Importantly, the PID gains and performance reported in this section were not treated as the final validated control system. Subsequent stages introduced:

1. documented first-order elevator-actuator dynamics;
2. elevator travel and rate constraints;
3. requested-versus-delivered actuator analysis;
4. reference-command shaping;
5. restoration of the thrust-control channel for forward-speed validity;
6. robustness and uncertainty assessment.

The final classical benchmark is therefore the later reference-shaped, actuator-constrained PID evaluated with the corrected longitudinal architecture, rather than the ideal-actuator PID presented in this section.

5. MATLAB/Simulink Model Validation

5.1 Validation Objective

MATLAB and Simulink were both used extensively during controller development.

MATLAB provided a convenient environment for:

- state-space analysis,
- pole and eigenvalue calculations,
- controller design,
- gain studies,
- time-domain simulation,
- quantitative performance calculations.

Simulink was used to construct explicit signal-flow implementations of the aircraft and controllers and later to introduce actuator dynamics, rate limiting, travel saturation, and disturbances. Because the two environments were used for different stages of the project, quantitative cross-validation was performed before relying on the Simulink models for subsequent engineering conclusions.

The objective was to verify that differences observed after introducing controllers or actuator nonlinearities resulted from the intended model changes rather than from inconsistent MATLAB and Simulink implementations.

5.2 Baseline Aircraft Simulink Model

The validated Boeing 747 longitudinal plant was transferred into Simulink using a State-Space block. Rather than manually entering a second set of numerical coefficients, the Simulink model referenced the same MATLAB workspace variables used by the validated aircraft model: A , B_e , C_θ , D_θ .

The baseline Simulink signal path consisted of the elevator input, the required unit conversion, the Boeing 747 State-Space block, and the pitch-angle output. The same elevator step used for the MATLAB open-loop analysis was applied in Simulink. The same -1° elevator step used for the open-loop MATLAB analysis was applied in Simulink. This ensured that both environments were evaluated using an identical aircraft model, input, initial condition, and simulation interval.

5.3 Numerical Comparison Method

The Simulink model used a variable-step numerical solver. Consequently, its output time samples were not identical to the uniformly specified MATLAB reference time vector. A direct point-by-point subtraction of the raw outputs would therefore not provide a valid comparison.

Instead, the validation procedure:

1. loaded the frozen Boeing 747 aircraft model;
2. executed the Simulink simulation;

3. generated the corresponding MATLAB reference response;
4. aligned both solutions on a common time vector;
5. interpolated the variable-step Simulink result where necessary;
6. calculated point-by-point pitch-angle error;
7. evaluated maximum and root-mean-square disagreement.

The principal error quantities were $e_\theta(t) = \theta_{\text{Simulink}}(t) - \theta_{\text{MATLAB}}(t)$, the maximum absolute error, $e_{\max} = \max|e_\theta(t)|$, and the root-mean-square error, $e_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^N e_\theta^2(t_i)}$.

5.4 Baseline Aircraft Validation Results

For the 400-second open-loop aircraft simulation, the MATLAB and Simulink results showed extremely close agreement. The measured discrepancies were approximately $e_{\max} 6.31 \times 10^{-4}^\circ$ and $e_{\text{RMS}} 3.19 \times 10^{-5}^\circ$. Relative to the peak aircraft pitch response, these corresponded to approximately 0.0103% maximum relative error and 0.00052% RMS relative error.

The two response curves were effectively coincident at the scale of the aircraft motion.

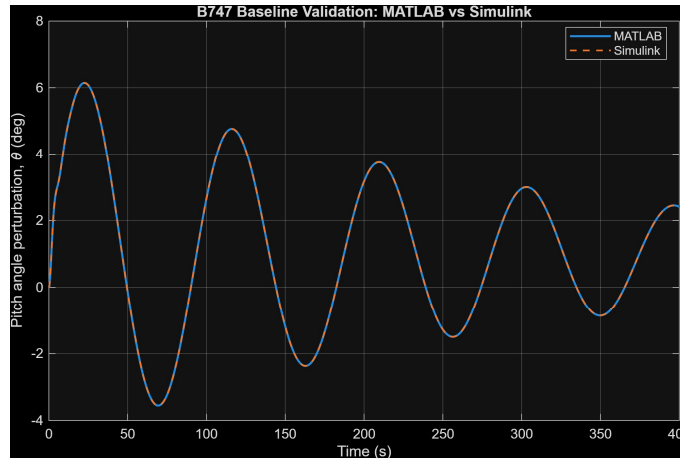


Figure 3. MATLAB and Simulink open-loop Boeing 747 pitch responses for the same elevator input. The nearly coincident responses demonstrate numerical consistency between the two implementations.

The small remaining discrepancy is negligible relative to the aircraft response and is consistent with numerical integration and interpolation differences.

The Simulink aircraft model was therefore accepted as a validated implementation of the MATLAB baseline plant.

5.5 PID Implementation Validation

The filtered PID architecture developed in Section 4 was also implemented explicitly in Simulink.

Rather than using a single preconfigured PID block, the model contained separate:

- proportional error feedback,
- integral error feedback,
- filtered derivative feedback from measured pitch,
- controller-output summation,
- aircraft state-space dynamics.

This made the implemented control law directly traceable to the MATLAB controller formulation. The MATLAB and Simulink PID pitch responses were again aligned on a common time vector and compared point by point. The maximum absolute pitch discrepancy was approximately $1.85 \times 10^{-3}^\circ$ and the RMS discrepancy was approximately $1.36 \times 10^{-4}^\circ$.

Relative to the peak pitch response, the maximum disagreement was approximately 0.0310% with RMS relative disagreement of approximately 0.00228%. These errors are negligible relative to the aircraft pitch response.

The MATLAB analytical controller model and explicit Simulink block-diagram implementation were therefore considered numerically consistent.

5.6 LQI Architecture Validation

A similar MATLAB–Simulink comparison was performed when the LQI architecture was introduced later in the project. For the validated LQI implementation, the maximum pitch-angle discrepancy was approximately $6.15 \times 10^{-8}^\circ$ with RMS pitch error of approximately $8.19 \times 10^{-9}^\circ$. The maximum discrepancy in the requested elevator command was approximately 5.09×10^{-8} , while the maximum discrepancy in actual first-order-actuator elevator response was approximately $6.72 \times 10^{-6}^\circ$. These differences are essentially numerical.

This validation confirmed the correctness of the LQI Simulink signal architecture before the subsequent structured LQI weight-tuning study. The final tuned LQI configuration was later evaluated using the complete actuator-constrained Simulink model and independently checked again during the final system-validation phase.

5.7 Reproducibility

Dedicated MATLAB scripts were developed to automate the principal numerical validation procedures.

The workflow separates the validated aircraft model, controller parameters, actuator assumptions, and analysis scripts so that key results can be regenerated from stored configurations rather than depending on manually retained MATLAB workspace variables.

The baseline aircraft is stored as `B747_longitudinal_baseline.mat` and the final controller and actuator configuration is stored separately as `final_control_configuration.mat`.

The MATLAB–Simulink comparison scripts independently validated the original baseline aircraft implementation, the classical PID signal architecture, and the LQI signal architecture.

Later final-validation studies were performed using dedicated MATLAB scripts that explicitly reconstructed the required final architecture, including:

- the restored thrust-control channel;

- forward-speed augmentation;
- PID reference shaping;
- actuator travel and rate constraints;
- multiple command profiles;
- internal-state monitoring;
- frequency-domain robustness analysis;
- repeated numerical uncertainty simulations.

Where intermediate results were required by later plotting or analysis scripts, results were saved explicitly to project data files to reduce dependence on transient workspace state.

This workflow provides traceability from the documented aircraft model through controller development and final engineering validation.

5.8 Validation Outcome

The MATLAB and Simulink implementations showed negligible numerical disagreement for the baseline aircraft, the classical PID architecture, and the LQI architecture tested during controller development.

These comparisons established that the fundamental state-space dynamics and controller signal paths were implemented consistently in the two software environments.

The later Phase 14 model-validity and robustness extensions were evaluated primarily in MATLAB and should therefore be distinguished from the earlier MATLAB–Simulink cross-validation. In particular, the final thrust/airspeed augmentation, reference-shaping study, frequency-domain analysis, multi-command validation, internal-state assessment, and Monte Carlo uncertainty study were not presented as separate MATLAB–Simulink equivalence tests.

The combined validation evidence therefore supports two distinct conclusions:

1. the original aircraft and controller implementations were numerically consistent between MATLAB and Simulink;
2. the final corrected architecture was subjected to additional independent numerical and engineering checks in MATLAB.

This distinction avoids overstating the scope of the software cross-validation while preserving its value in establishing confidence in the underlying aircraft and controller implementations.

6. Actuator Dynamics and Physical Constraints

6.1 Motivation

The classical controller development in Section 4 initially assumed that commanded elevator motion could be applied directly to the aircraft. A real elevator actuator cannot respond instantaneously and is subject to limits on both control-surface travel and deflection rate.

To increase the physical realism of the simulation, a documented Boeing 747-100/200 elevator-actuator representation was introduced and the previously selected PID controller was reevaluated.

Three actuator effects were considered:

- first-order actuator dynamics,
- finite elevator travel,
- finite elevator rate.

This step was necessary because satisfactory aircraft pitch response does not automatically imply that the required control-surface motion is physically achievable.

6.2 Elevator Actuator Model

The elevator actuator was modeled as the first-order transfer function $A(s) = \frac{\delta_e(s)}{\delta_{e,cmd}(s)} = \frac{37}{s+37}$. The corresponding time constant is $\tau_a = \frac{1}{37} = 0.0270$ s. The adopted elevator travel limits were $-23^\circ \leq \delta e \leq +17^\circ$, with a maximum deflection-rate magnitude of $|\dot{\delta}_e| \leq \frac{37^\circ}{s}$. These parameters were adopted from a documented B747-100/200 actuator representation.

The actuator source is separate from the MIT Boeing 747 longitudinal state-space source used for the baseline aircraft dynamics. The actuator values are therefore treated as documented B747 actuator constraints adopted for this study rather than as a claim that both source models represent the identical aircraft configuration.

6.3 Effect of First-Order Actuator Lag

The actuator transfer function was first introduced without travel or rate saturation.

The ideal-elevator and first-order-actuator responses were:

Table 3. Ideal-Elevator and First-Order-Actuator Performance Comparison

Metric	Ideal Elevator	First-Order Actuator
Final Pitch	5.0000°	5.0000°
Maximum Pitch	5.9750°	5.9895°
Overshoot	19.50%	19.79%
Peak time	4.02 s	3.98 s
2% settling time	25.57 s	25.48 s

The maximum point-by-point pitch-response difference was only 0.0465°. Thus, actuator lag alone produced only a minor change in pitch response. This is consistent with the short actuator time constant, $\tau_a \approx 0.027$ s, which is much faster than the dominant aircraft longitudinal dynamics.

6.4 Initial Actuator Feasibility Audit

Although the pitch response changed very little, the physical elevator motion revealed a substantially different result. For the nominal $0^\circ \rightarrow 5^\circ$ pitch command, the unconstrained first-order actuator reached approximately $\delta e_{min} = -4.1368^\circ$ and $\delta e_{max} \approx 0^\circ$. These values were comfortably within the adopted travel limits. Therefore, Elevator travel: PASS. However, the predicted maximum actuator rate was approximately $148^\circ/\text{s}$. Compared with the adopted physical rate limit, $37^\circ/\text{s}$, the unconstrained model demanded approximately $\frac{148}{37} = 4$ times the available rate capability. Therefore, Elevator rate: FAIL

This was a significant result: examining pitch angle alone would have suggested that the actuator model had little effect, while direct inspection of surface motion exposed a genuine feasibility problem.

6.5 Nonlinear Actuator Model

The Simulink model was therefore expanded to include the complete actuator chain:

$$\text{PID} \rightarrow \frac{37}{s + 37} \rightarrow \text{Rate Limiter} \rightarrow \text{Travel Saturation} \rightarrow \text{Aircraft}$$

The actuator constraints were $-\frac{37^\circ}{s} \leq \dot{\delta}_e \leq \frac{37^\circ}{s}$ and $-23^\circ \leq \delta_e \leq 17^\circ$. Both the controller-requested elevator and the actual actuator output were recorded so that command demand and physically delivered control motion could be evaluated separately.

6.6 Original PID Under Physical Constraints

The PID architecture developed during the ideal-actuator phase was next evaluated using the nonlinear actuator model.

The actuator model included:

- first-order actuator dynamics;
- elevator travel limits of -23° and $+17^\circ$;
- a maximum elevator rate magnitude of $37^\circ/\text{s}$.

The initial nonlinear simulations demonstrated that actuator-rate limiting could materially alter the closed-loop response even when elevator travel remained well within its allowable range.

This established that pitch response alone was insufficient for assessing controller feasibility. Both the controller-requested elevator motion and the physically delivered actuator motion had to be examined directly.

6.7 Final PID Architecture

After subsequent model-validity reassessment, the final classical controller retained the filtered PID gains $K_p = 1$, $K_i = 0.5$, $K_d = 0.8$, $T_f = 0.1$ s.

The final PID was evaluated using the corrected longitudinal architecture containing:

- the nonlinear elevator actuator;
- the restored thrust-control channel;

- forward-speed augmentation;
- explicit requested-versus-delivered actuator monitoring.

Before command shaping was introduced, a nominal 5° pitch step generated an initial requested elevator-rate magnitude of approximately 185.1°/s.

This is approximately 5.0 times the adopted actuator-rate capability of 37°/s.

Although the nonlinear rate limiter prevented the physical actuator from exceeding its modeled limit, relying on severe rate saturation is undesirable because it causes the implemented control action to differ substantially from the controller request.

A reference-command shaper was therefore introduced.

6.8 Reference-Command Shaping

The pitch command was passed through a first-order reference model, θ_{cmd} , where θ_{cmd} is the original command and $\theta_{cmd,f}$ is the command supplied to the PID tracking loop.

A sweep of reference-shaper time constants was performed to identify the fastest command shaping that avoided excessive actuator-rate demand.

The evaluated time constants were $\tau_r = 0.05, 0.10, 0.15, 0.20, 0.30, 0.40, 0.50$ s.

The fastest candidate that kept the requested actuator rate below a 90% utilization target of $0.9(37) = 33.3^\circ/\text{s}$ was $\tau_r = 0.10$ s.

For this configuration, the nominal maximum requested elevator rate was approximately 31.57°/s, corresponding to approximately 85.3% of the available actuator-rate capability.

The requested and physically delivered actuator rates were effectively identical, indicating that the rate limiter no longer dominated the nominal maneuver.

The 0.10 s shaper was therefore frozen as part of the final PID architecture.

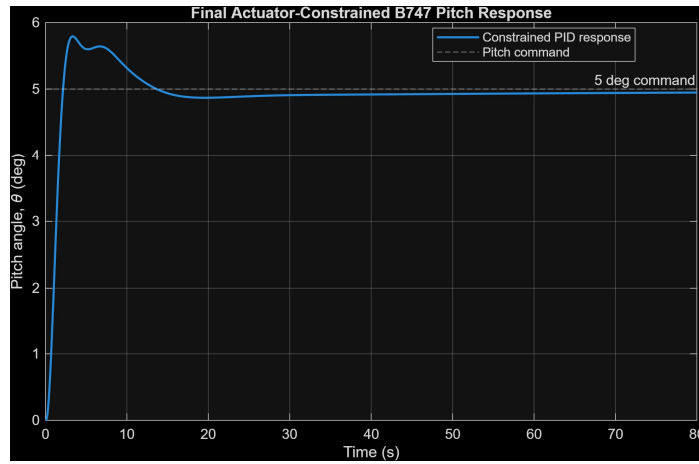


Figure 4. Nominal 5° pitch response of the final reference-shaped PID using the corrected longitudinal architecture and nonlinear elevator actuator. The controller satisfies the adopted overshoot, settling-time, elevator-travel, and actuator-rate criteria.

6.9 Final Reference-Shaped PID Performance

Using the final reference-shaped PID, corrected thrust/airspeed augmentation, and nonlinear actuator model, the nominal 5° command produced:

- final pitch angle: approximately 5.000°;
- overshoot: 15.33;
- 2% settling time: 14.07 s;
- maximum requested elevator rate: 31.57°/s;
- maximum actual elevator rate: 31.57°/s;
- maximum elevator magnitude: approximately 4.68°;
- maximum forward-speed perturbation: approximately 0.53 m/s;
- maximum forward-speed deviation relative to trim: approximately 0.227

The maneuver therefore satisfied the adopted project overshoot, settling-time, elevator-travel, and elevator-rate criteria.

Unlike the earlier constrained configuration, the final design does not depend on the actuator rate limiter to enforce feasibility under the nominal command.

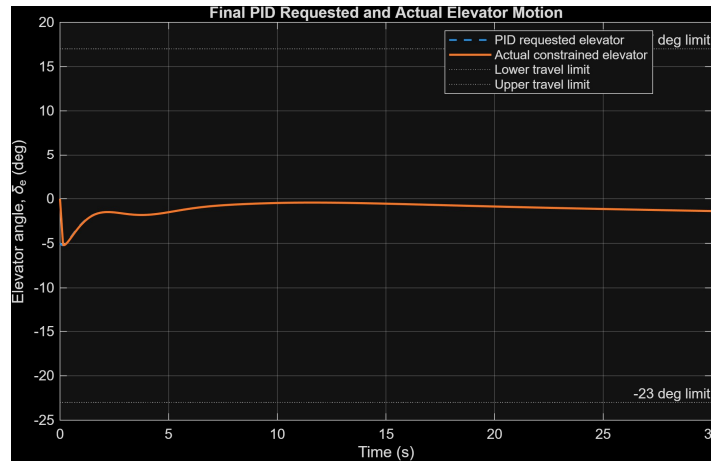


Figure 5. Requested and delivered elevator motion for the final reference-shaped PID architecture. The 0.10 s command shaper reduces the requested actuator rate below the adopted 37°/s limit, allowing the delivered elevator response to closely follow the controller request without sustained rate saturation.

6.10 Reduced-Elevator-Effectiveness Check

The final PID was also evaluated with 20% reduced elevator effectiveness.

Under this condition, the controller produced approximately:

- overshoot: 16.23;
- 2% settling time: 15.09 s;
- maximum requested elevator rate: 31.58°/s;
- maximum actual elevator rate: 31.58°/s;
- maximum elevator magnitude: approximately 4.82°;
- maximum forward-speed perturbation: approximately 0.53 m/s.

The controller therefore remained stable, tracked the 5° command, and satisfied the adopted project transient and actuator criteria.

This is an important correction to the earlier elevator-only analysis, which had indicated a settling-time failure under reduced elevator effectiveness. Once the forward-speed-validity architecture and final reference shaping were included, the weak-elevator PID case satisfied the project performance requirements.

6.11 Engineering Interpretation

The actuator study demonstrates that physical feasibility should be evaluated at the controller-output level rather than inferred from aircraft pitch response alone.

The first-order actuator lag itself produced only a modest change in aircraft pitch response. In contrast, direct inspection of requested elevator motion revealed that an unshaped step command demanded substantially more actuator rate than the adopted physical limit.

The reference shaper solved this issue at the command level rather than relying on nonlinear actuator saturation.

For the final nominal PID maneuver, the maximum requested elevator rate was approximately $31.57^\circ/\text{s}$, corresponding to about 85.3% of the adopted $37^\circ/\text{s}$ actuator-rate capability.

The resulting architecture provides useful rate margin while preserving the desired transient performance.

This result also illustrates an important control-system design principle: actuator feasibility can sometimes be improved more effectively by shaping the commanded maneuver than by retuning the feedback controller itself.

7. Disturbance Rejection

7.1 Objective

The disturbance-rejection analysis evaluated the ability of the pitch-control system to maintain the commanded aircraft attitude when subjected to a temporary control-channel disturbance. The aircraft was commanded to maintain $\theta_{\text{cmd}} = 5^\circ$ while a temporary positive elevator disturbance was applied from $t = 40$ s to $t = 42$ s, giving a disturbance duration of $\Delta t_d = 2$ s. Under the adopted aircraft sign convention, positive elevator deflection represents elevator-down motion. Therefore, the imposed positive disturbance acts in a direction that temporarily drives the aircraft away from the commanded nose-up pitch condition.

An additive control-channel disturbance was selected because it provides a controlled and physically interpretable test of feedback rejection capability without requiring an additional aerodynamic gust model. The disturbance test should therefore be interpreted as a **control-channel disturbance-rejection study**, rather than as a complete representation of atmospheric turbulence or aerodynamic gust loading.

The disturbance-rejection study presented in this section was completed during the original actuator-constrained controller-development phase, before the later thrust/airspeed augmentation and final command-shaping architecture were introduced. The results are therefore retained as a controlled comparison of feedback disturbance rejection under the original actuator-aware architecture. They are not used as the principal final validation of the corrected Phase 14 longitudinal model.

7.2 Preliminary Unconstrained Disturbance Study

Disturbance rejection was first investigated before physical actuator constraints were introduced. The initial PID used in this preliminary study was $K_p = 0.8$, $K_I = 0.5$, $K_d = 0.8$, $T_f = 0.1$ s. Two disturbance amplitudes were evaluated: $\delta_{e,d} = +1^\circ$ and $\delta_{e,d} = +2^\circ$, with both disturbances applied over $40 \leq t < 42$ s. The total physical elevator input was represented as $\delta_{e,\text{total}}(t) = \delta_{e,\text{controller}}(t) + \delta_{e,\text{disturbance}}(t)$. Because the aircraft and controller models were linear at this stage, the total pitch response could also be interpreted using superposition: $\theta_{\text{total}}(t) = \theta_{\text{nominal}}(t) + \theta_{\text{disturbance}}(t)$. This allowed the incremental influence of the disturbance to be evaluated separately from the nominal pitch-command response.

7.3 Preliminary Disturbance Results

For the $\delta_{e,d} = +1^\circ$ disturbance, the maximum deviation from the commanded pitch was $|\theta - \theta_{\text{cmd}}|_{\text{max}} = 0.7436^\circ$, occurring at approximately $t = 42.05$ s. The disturbance-only contribution reached $|\theta_d|_{\text{max}} = 0.6640^\circ$. A recovery band corresponding to 2% of the 5° pitch command was defined as $5^\circ \pm 0.1^\circ$, or equivalently, $4.9^\circ \leq \theta \leq 5.1^\circ$. Following disturbance removal, the aircraft returned permanently to this band after approximately $t_{\text{rec}} = 4.80$ s. The maximum additional PID elevator action was $\Delta\delta_{e,\text{PID}} = 0.9853^\circ$, which corresponds to approximately $\eta_{\text{comp}} = \frac{0.9853}{1.0000} \times 100 = 98.53\%$ peak disturbance compensation.

When the disturbance magnitude was doubled to $\delta_{e,d} = +2^\circ$, the maximum pitch-command deviation increased to $|\theta - \theta_{\text{cmd}}|_{\text{max}} 1.4076^\circ$. The recovery time increased to $t_{\text{rec}} = 5.95$ s, and the maximum additional PID action increased to $|\Delta\delta_{e,\text{PID}}|_{\text{max}} 1.9706^\circ$. The preliminary disturbance results are summarized below.

Table 4. Preliminary Disturbance-Rejection Results

Metric	+1° Disturbance	+2° Disturbance
Disturbance duration	2.00 s	2.00 s
Maximum pitch deviation	0.7436°	1.4076°
Time of maximum deviation	42.05 s	42.05 s
Recovery band	$\pm 0.100^\circ$	$\pm 0.100^\circ$
Recovery time	4.80 s	5.95 s
Maximum additional PID action	0.9853°	1.9706°
Peak disturbance compensation	98.53%	98.53%
Closed-loop response	Stable	Stable

Doubling the disturbance approximately doubled the required incremental controller action: $1.9706^\circ \approx 2(0.9853^\circ)$. This behavior is consistent with the linear aircraft and controller model used during the preliminary disturbance study. The result should therefore be interpreted as confirmation of approximately linear disturbance-response scaling rather than as evidence of robustness to nonlinear aircraft behavior or model uncertainty.

7.4 Actuator-Constrained Disturbance Model

The preliminary analysis did not yet establish physical actuator feasibility. After the realistic actuator model was introduced, disturbance rejection was repeated using the final actuator-aware PID: $K_P = 1.0$, $K_I = 0.5$, $K_D = 0.8$, $T_f = 0.1$ s. The actuator dynamics were $A(s) = \frac{37}{s+37}$, with travel limits $-23^\circ \leq \delta_e \leq 17^\circ$, and rate limit $|\dot{\delta}_e| \leq \frac{37^\circ}{\text{s}}$. The constrained controller signal path was PID $\rightarrow \frac{37}{s+37} \rightarrow$ Rate Limiter $\rightarrow$ Travel Saturation $\rightarrow$ Aircraft. The additive disturbance was introduced at the aircraft control input after the actuator.

Therefore, the actuator constraints apply to the **corrective elevator motion generated by the controller**, while the disturbance represents an external additive control-channel perturbation.

7.5 Constrained +1° Disturbance

For the constrained disturbance $\delta_{e,d} = +1^\circ$, the maximum pitch deviation from the 5° command was approximately $|\theta - \theta_{\text{cmd}}|_{\text{max}} 0.7051^\circ$. The largest deviation occurred at approximately $t = 42.03$ s. The disturbance-only contribution reached approximately $|\theta_d|_{\text{max}} 0.6254^\circ$. Following disturbance removal, the aircraft returned permanently to the $5^\circ \pm 0.1^\circ$ recovery region after approximately $t_{\text{rec}} 4.14$ s.

The physical actuator remained within both its travel and rate limits. Therefore, Travel Limit: PASS and Rate Limit: PASS.

7.6 Constrained +2° Disturbance

The stronger final disturbance case used $\delta_{e,d} = +2^\circ$. From $40 \leq t < 42$ s. The maximum deviation of the complete aircraft response from the 5° command was $|\theta - \theta_{\text{cmd}}|_{\text{max}} 1.3305^\circ$. The isolated disturbance contribution reached $|\theta_d|_{\text{max}} 1.2507^\circ$. The aircraft returned permanently to the $5^\circ \pm 0.1^\circ$ recovery region after approximately $t_{\text{rec}} 5.4855$ s. The closed-loop aircraft remained stable throughout the disturbance and subsequent recovery.

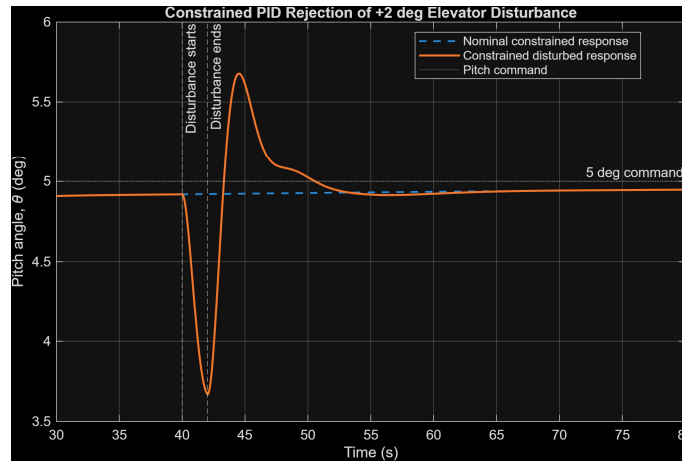


Figure 6. Actuator-constrained pitch response to a +2° two-second additive control-channel disturbance. The closed-loop aircraft remains stable and returns to the commanded pitch following disturbance removal.

This disturbance result corresponds to the developmental actuator-aware architecture and is retained for controller-response assessment rather than final corrected-model qualification.

7.7 Actuator Demand During Disturbance Rejection

The overall maximum actuator rate during the complete +2° simulation was $\frac{|\dot{\delta e}|_{\max 37^\circ}}{s}$. However, this maximum occurred during the initial 5° pitch-command maneuver rather than during the disturbance itself. During the disturbance and subsequent recovery period, the maximum actuator rate was only approximately $\frac{|\dot{\delta e}|_{\max, \text{dist}} .0757^\circ}{s}$. Relative to the available actuator-rate capability, $\frac{2.0757}{37} \times 100 \approx 5.61\%$

Thus, rejection of the tested +2° disturbance required only approximately **5.61% of the available elevator-rate capability** once the aircraft was near its commanded operating condition. Both actuator checks remained satisfied: Travel Limit: PASS, Rate Limit: PASS.

7.8 Preliminary and Constrained Comparison

Within this developmental disturbance study, the actuator-constrained case is treated as the principal disturbance-rejection result because it includes the physical elevator actuator model. The earlier unconstrained analysis remains useful for demonstrating the controller's opposing action and approximately linear response to increasing disturbance magnitude.

Table 5. Preliminary and Actuator-Constrained Disturbance Comparison

Metric	Preliminary PID	Final Constrained PID
Kp	0.8	1.0
Ki	0.5	0.5
Kd	0.8	0.8
Disturbance amplitude	+2°	+2°
Disturbance duration	2.00 s	2.00 s
Maximum total pitch deviation	1.4076°	1.3305°
Disturbance-only pitch peak	—	1.2507°
Recovery time	5.95 s	5.4855 s
Actuator constraints included	No	Yes
Closed-loop response	Stable	Stable

The constrained result is treated as the primary engineering result because it uses the final actuator-aware PID and includes the physical actuator model. The earlier unconstrained analysis remains useful for demonstrating the controller's opposing action and approximately linear response to increasing disturbance magnitude.

7.9 Engineering Interpretation

The disturbance study was intentionally developed in stages.

The preliminary linear analysis demonstrated that the PID controller generated corrective elevator action opposing the imposed control-channel disturbance and that increasing disturbance magnitude produced approximately proportional increases in pitch deviation and corrective control demand.

The actuator-constrained analysis then established that the selected short-duration disturbance could be rejected without violating the adopted elevator travel or rate limits. For the final disturbance case evaluated in this section, the actuator-aware PID limited the disturbance-only pitch excursion to approximately 1.2507° and returned the aircraft to the selected recovery band in approximately 5.49 s.

The tested disturbance therefore provides useful evidence of closed-loop disturbance-rejection capability within the original actuator-aware simulation architecture.

However, this section predates the later thrust/airspeed-validity correction and final reference-shaping architecture. Its numerical disturbance results are consequently retained as developmental evidence rather than as a complete final-system qualification.

The disturbance itself is also deliberately limited in scope. It is an additive control-channel perturbation and does not explicitly represent atmospheric turbulence, vertical gusts, wind shear, sensor disturbances, structural excitation, or engine disturbances.

The appropriate conclusion is therefore that the developed feedback controller demonstrated effective rejection of the specific modeled control-channel disturbance while remaining within the adopted elevator constraints. Broader aircraft disturbance qualification would require physically defined aerodynamic gust and turbulence models evaluated using the final corrected longitudinal architecture.

8. Robustness and Uncertainty Analysis

8.1 Objective

The final robustness assessment evaluated the sensitivity of the corrected longitudinal-control architecture to uncertainty in the aerodynamic aircraft model.

Earlier development studies varied individual parameters such as elevator effectiveness and pitch-rate damping. Those studies were useful for identifying potentially important sensitivities, but they examined only a limited number of isolated or corner cases.

The final robustness study therefore used a broader repeated numerical uncertainty analysis in which multiple selected aerodynamic coefficients were varied simultaneously.

Both the final reference-shaped PID and the tuned LQI controllers were evaluated using the same uncertain aircraft realization for each trial.

No controller gains were retuned between cases.

8.2 Final Uncertainty Model

The uncertainty study was performed using 500 reproducible numerical trials.

For each trial, selected nonzero aerodynamic entries of the longitudinal state matrix A were independently perturbed using uniformly distributed multiplicative variations of $\pm 10\%$.

The exact gravity and kinematic relationships, $A_{14} = -g$ and $A_{43} = 1$, were not perturbed because they are not aerodynamic coefficients.

The nonzero elevator-input coefficients in B_e were also independently varied by $\pm 10\%$.

The same perturbed aircraft realization was used for both PID and LQI in each trial to provide a controlled controller comparison.

The thrust/airspeed augmentation, actuator model, PID reference shaper, controller gains, and project evaluation criteria remained fixed throughout the study.

The random-number generator was initialized using a fixed seed so that the entire 500-case study could be reproduced.

8.3 Interpretation of the Uncertainty Distribution

The selected $\pm 10\%$ independent uniform perturbations are engineering stress-test assumptions rather than experimentally identified Boeing 747 uncertainty statistics.

The study therefore does not establish statistical probabilities of real-aircraft behavior.

In particular, the selected coefficient variations do not represent:

- measured covariance between aerodynamic derivatives;
- configuration-specific flight-test uncertainty;
- explicit center-of-gravity variation;
- explicit altitude or Mach-number variation;
- a certified flight-envelope uncertainty model.

The Monte Carlo results should therefore be interpreted as controller behavior over a reproducible numerical perturbation set rather than as a probabilistic reliability prediction.

8.4 Robustness Evaluation Criteria

Each simulation was evaluated using the same principal project-level criteria: $M_p \leq 20\%$ and $t_s \leq 30$ s.

Additional checks included:

- successful convergence to the commanded pitch;
- actuator-rate feasibility;

- forward-speed departure;
- approximate angle-of-attack excursion;
- numerical stability of the simulation.

The uncertainty analysis therefore distinguished between:

1. closed-loop stability and command tracking;
2. transient-performance compliance;
3. actuator feasibility;
4. maintenance of reasonable internal-state excursions.

8.5 PID Uncertainty Results

All 500 reference-shaped PID simulations completed successfully and tracked the commanded pitch.

The overshoot distribution produced:

- median overshoot: 15.74%;
- 95th-percentile overshoot: 20.07%;
- maximum observed overshoot: 21.58%.

The settling-time results were:

- median: 14.07 s;
- 95th percentile: 15.16 s;
- maximum observed: 16.08 s.

The requested elevator-rate demand remained highly consistent across the uncertainty set:

- median maximum requested rate: $31.87^\circ/\text{s}$;
- 95th percentile: $31.87^\circ/\text{s}$;
- maximum observed: $31.87^\circ/\text{s}$.

All PID trials therefore remained below the adopted $37^\circ/\text{s}$ actuator-rate limit.

The maximum forward-speed perturbation relative to trim remained small:

- median: 0.226%;
- 95th percentile: 0.258%;
- maximum observed: 0.275%.

The approximate angle-of-attack perturbation remained:

- median: 3.83°;
- 95th percentile: 4.13°;
- maximum observed: 4.24°.

Of the 500 PID trials, 475 satisfied all defined project criteria.

The remaining 25 cases exceeded only the 20% overshoot threshold. None became unstable, failed to track the pitch command, violated the actuator-rate limit, or produced excessive forward-speed departure under the adopted screening criteria.

Thus, the final PID demonstrated strong but not universal transient-performance compliance over the selected numerical uncertainty set.

8.6 LQI Uncertainty Results

All 500 LQI simulations also completed successfully and tracked the commanded pitch.

The overshoot distribution was substantially lower than for PID:

- median overshoot: 0.717%;
- 95th percentile: 1.263%;
- maximum observed overshoot: 1.526%.

The settling-time results were:

- median: 5.588 s;
- 95th percentile: 5.730 s;
- maximum observed: 5.784 s.

Peak requested elevator-rate demand remained low:

- median: 5.334°/s;
- 95th percentile: 5.370°/s;
- maximum observed: 5.374°/s.

The forward-speed deviation remained:

- median: 0.194%;
- 95th percentile: 0.223%;
- maximum observed: 0.239%.

The approximate angle-of-attack perturbation remained:

- median: 2.78°;

- 95th percentile: 3.05°;
- maximum observed: 3.17°.

All 500 LQI trials satisfied every defined project criterion.

This result demonstrates substantially greater performance consistency than the PID over the selected perturbation set.

8.7 Summary of Numerical Uncertainty Results

The principal uncertainty statistics are summarized below.

Table 6. Final 500-Trial Numerical Uncertainty Results

Metric	PID	LQI
Successful/tracked cases	500/500	500/500
Cases satisfying all project criteria	475/500	500/500
Median overshoot	15.74%	0.717%
95th-percentile overshoot	20.07%	1.263%
Worst observed overshoot	21.58%	1.526%
Median settling time	14.07 s	5.588 s
95th-percentile settling time	15.16 s	5.730 s
Worst observed settling time	16.08 s	5.784 s
Median requested actuator rate	31.87°/s	5.334°/s
Maximum requested actuator rate	31.87°/s	5.374°/s
95th-percentile forward-speed deviation	0.258%	0.223%
Worst approximate $ \Delta\alpha $	4.24°	3.17°

The results show that both controllers remained dynamically well behaved throughout the numerical uncertainty study.

However, the LQI exhibited much lower variation in transient performance and maintained large margins relative to the project limits.

8.8 Relation to the Earlier Parameter Studies

The earlier isolated elevator-effectiveness and pitch-damping studies remain useful as developmental sensitivity analyses.

For example, the original elevator-only architecture produced a 40.73 s settling time under 20% reduced elevator effectiveness. That result led to the initial conclusion that reduced control effectiveness was the dominant PID limitation.

After the thrust/airspeed correction and reference shaping were introduced, however, the same 20% reduced elevator-effectiveness case produced approximately 16.23% overshoot and 15.09 s settling time.

The case therefore satisfied both project transient criteria.

This comparison demonstrates that uncertainty conclusions depend on the complete control-system architecture. The earlier weak-elevator performance degradation was not solely a property of the PID gains; it was also influenced by the original elevator-only model architecture.

The final Monte Carlo assessment supersedes the earlier corner-study conclusions for the final system.

8.9 Robust Stability and Performance Interpretation

Every one of the 500 numerical trials for both controllers remained stable and successfully tracked the commanded pitch.

Within the tested numerical uncertainty set:

- PID showed no instability;
- LQI showed no instability;
- neither controller violated the adopted actuator-rate constraint;
- both maintained small forward-speed departures relative to the trim speed.

The study nevertheless preserves an important distinction between robust stability and robust performance.

Although the PID remained stable in all 500 cases, 25 trials exceeded the project overshoot target.

Thus, stability across all tested cases does not imply that all transient-performance requirements are satisfied across all tested cases.

For the LQI, all 500 tested cases satisfied both stability and the defined project performance criteria.

These observations apply only to the selected uncertainty model and should not be interpreted as a formal mathematical proof of robust stability for all possible Boeing 747 parameter variations.

8.10 Engineering Interpretation

The final uncertainty analysis strengthens the controller comparison in several ways.

First, it evaluates simultaneous variation of multiple aerodynamic coefficients rather than relying only on isolated one-parameter sweeps.

Second, both controllers are tested against identical uncertain aircraft realizations, avoiding bias caused by comparing different perturbation sets.

Third, the results show that the final thrust/airspeed augmentation substantially improves the physical consistency of the uncertainty simulations by keeping forward-speed departure small.

The reference-shaped PID remains a credible classical controller. It successfully tracked the commanded pitch in every tested case, remained within the actuator-rate limit, and satisfied all defined criteria in 475 of 500 trials.

The LQI showed greater performance robustness. It satisfied all defined criteria in all 500 numerical trials while maintaining substantially lower overshoot, shorter settling time, and lower actuator demand.

The result does not establish real-world probability of success, because the uncertainty distributions were selected as reproducible engineering stress-test assumptions rather than experimentally derived statistical distributions.

The appropriate conclusion is therefore that, over the selected numerical aerodynamic-uncertainty set, the LQI demonstrated stronger and more consistent transient performance than the PID while both controllers retained stable command tracking.

9. Advanced State-Feedback Control

9.1 Objective

The classical PID controller developed in the preceding sections provided stable and physically feasible pitch control. Earlier developmental studies showed sensitivity to reduced elevator effectiveness, while the later corrected architecture improved that behavior substantially. An advanced state-feedback controller was therefore investigated to determine whether further improvements in transient performance, actuator demand, and robustness could be achieved.

An advanced state-feedback controller was therefore investigated to determine whether improved transient performance, disturbance rejection, and reduced sensitivity to control-effectiveness changes could be achieved.

Because the Boeing 747 longitudinal aircraft was already represented in state-space form, $\dot{x} = Ax + B_e \delta_e$, Linear Quadratic Regulator (LQR) control was selected as the starting point.

The analysis was subsequently extended to **Linear Quadratic Integral (LQI)** control to provide accurate tracking of nonzero pitch commands.

The final advanced controller was evaluated using the same:

- aircraft model,
- 5° pitch command,
- first-order actuator dynamics,
- elevator travel limits,
- elevator rate limit,

- $+2^\circ$ disturbance,
- 20% reduced-elevator-effectiveness case

used for the final PID evaluation.

9.2 Controllability of the Longitudinal Aircraft

The longitudinal state vector is $x = [u \ w \ q \ \theta]^T$, with elevator deflection δ_e as the single control input. Before designing a full-state feedback controller, the controllability of the aircraft model was evaluated. For a four-state system, the controllability matrix is $\mathcal{C}[B_e \ AB_e \ A^2B_e \ A^3B_e]$. The calculated controllability rank was $\text{rank}(\mathcal{C}) = 4$. Since $\text{rank}(\mathcal{C}) = n$, where $n = 4$, the aircraft is fully controllable through the elevator input at the selected operating condition. Therefore, Longitudinal Aircraft: Fully Controllable and full-state feedback design is mathematically feasible.

9.3 Initial LQR Design

The LQR controller determines a state-feedback law of the form $\delta_e = -Kx$ by minimizing the quadratic performance index $J = \int_0^\infty (x^T Q x + \delta_e^T R \delta_e) dt$. The matrix (Q) penalizes deviations of the aircraft states, while (R) penalizes control effort. Rather than selecting arbitrary numerical weights, the initial weighting matrices were constructed using physically interpretable state and actuator normalization scales. The approximate initial LQR gain was $K = [0.0009 \ 0.0006 \ -0.2601 \ -0.2779]$. The resulting closed-loop poles were approximately $-0.0501 \pm 0.0671i$ and $-0.4736 \pm 0.9211i$. All poles have negative real parts. Therefore, Initial LQR Regulation: Stable. The LQR design successfully stabilized and regulated deviations of the longitudinal states toward zero. However, regulation about the trim condition is not equivalent to accurate tracking of a nonzero pitch-angle command.

9.4 Static Reference-Prefilter Limitation

To allow the LQR regulator to track the commanded pitch angle, $\theta_{\text{cmd}} = 5^\circ$, a static reference prefilter, N_{bar} , was initially introduced. The resulting prefilter was approximately $N_{\text{bar}} = -1.9457$. This produced a command-to-pitch DC gain of unity, meaning that the system had the correct theoretical steady-state tracking gain. However, the 5° reference step generated an immediate elevator command of approximately -9.73° .

The resulting aircraft response reached approximately $\theta_{\text{max}} = 21.51^\circ$, corresponding to approximately 330% overshoot. The settling time was approximately 118.5 s. Thus, although the state-feedback system was stable and had correct steady-state gain, stable regulation $\neq$ satisfactory command tracking. The static-reference LQR architecture was therefore rejected for the pitch-command problem. A servo architecture containing integral tracking action was introduced instead.

9.5 LQI Formulation

An integral state was defined from the pitch tracking error: $\xi = \theta_{\text{cmd}} - \theta$.

The augmented state vector became $\mathbf{x}_a = \begin{bmatrix} \mathbf{x} \\ \xi \end{bmatrix} = \begin{bmatrix} u \\ w \\ q \\ \theta \\ \xi \end{bmatrix}$

The resulting LQI control law was $\delta_e = -K_x \mathbf{x} - K_\xi \xi$.

The augmented state-space system can be written as $\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C_\theta & 0 \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix} + \begin{bmatrix} B_e \\ 0 \end{bmatrix} \delta_e + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \theta_{cmd}$. The augmented system contains five states. Its controllability rank was $\text{rank}(C_a) = 5$. Therefore, Augmented LQI System: Fully Controllable. Integral action provides the controller with the ability to generate the sustained elevator bias required for zero steady-state pitch-command error.

9.6 Initial LQI Design

The first LQI design used conservative state, integral-error, and elevator-effort weighting. The resulting state-feedback gains were approximately $K_x = [-0.0005 \quad 0.0016 \quad -0.4656 \quad -0.6716]$ with integral gain $K_\xi = 0.1$. The augmented closed-loop poles were approximately -0.0111 , $-0.1380 \pm 0.1092i$, and $-0.4964 \pm 0.9380i$. All poles had negative real parts. For the nominal 5° pitch command, the initial LQI design produced approximately 0.67% overshoot and 17.87 s settling time. When the first-order actuator was included, the maximum actuator rate was only approximately $0.48^\circ/\text{s}$, well below the adopted limit of $37^\circ/\text{s}$. The initial LQI therefore produced dramatically smoother nominal control action than the actuator-aware PID.

9.7 Long-Duration Equilibrium Validation

The LQI design contains a relatively slow closed-loop mode near -0.0111 . Its corresponding approximate time constant is $\tau \approx \frac{1}{0.0111} \approx 90$ s. A long-duration simulation was therefore performed to verify the final equilibrium rather than relying only on a shorter transient simulation. The aircraft DC gain independently predicts that maintaining a 5° pitch perturbation requires approximately $\delta_{e,ss} = 5.417^\circ$. At the end of the extended simulation, the LQI elevator command converged to approximately -5.416° , in excellent agreement with the independent equilibrium prediction. This confirmed that the apparent slow evolution of elevator demand was associated with the slow closed-loop mode rather than with incorrect integral action.

9.8 Initial LQI Limitations

Although the initial LQI produced excellent nominal tracking, additional testing revealed weaknesses that were not apparent from the nominal command response alone. For the final actuator-aware PID, the tested $+2^\circ$ disturbance produced a disturbance-only pitch peak of 1.2507° with recovery in approximately 5.49 s. The initial conservative LQI produced a larger disturbance response and required approximately 14.26 s to recover. A second weakness appeared when elevator effectiveness was reduced by 20%. The initial LQI remained stable and ultimately returned to the commanded pitch, but its settling time increased to approximately 100.9 s. Long-duration analysis confirmed that the correct equilibrium was still reached.

Therefore, the degraded behavior was caused primarily by **overly conservative controller weighting**, rather than by:

- instability,
- actuator saturation,
- insufficient steady-state control authority.

This result demonstrated that excellent nominal response alone was not sufficient for selecting the final LQI design.

9.9 Structured LQI Weight Tuning

A structured LQI weight study was therefore performed.

The normalization of the physical aircraft states was held fixed while two interpretable design quantities were varied:

- integral-error normalization scale,
- elevator-effort normalization scale.

The integral-error scale was evaluated at 10°, 7.5°, 5°, 2.5°, while the elevator-effort scale was evaluated at 1°, 1.5°, 2°, 3°. The combination produced $4 \times 4 = 16$ candidate LQI designs.

Each candidate was evaluated using the same principal engineering tests:

- nominal 5° command tracking,
- 20% reduced elevator effectiveness,
- +2° disturbance rejection,
- actuator-rate demand,
- elevator travel feasibility,
- closed-loop stability.

This changed the LQI design process from tuning for nominal response alone to selecting a controller using **multiple performance and robustness criteria simultaneously**.

9.10 Final Tuned LQI Selection

The structured weighting study produced a final LQI design that provided strong command tracking while maintaining low actuator demand.

The final frozen full-state feedback gains were $K_x = [-0.0004253 \quad 0.0022330 \quad -1.6776694 \quad -2.6675019]$ with integral gain $K_\xi = 1.2$.

The implemented control law was $\delta_e = -K_x x - K_\xi \xi$, where the integral state satisfies $\dot{\xi} = \theta - \theta_{cmd}$.

The corresponding frozen augmented-design poles were approximately -0.01133 , -0.32976 , -0.56491 , and $-0.88629 \pm 1.18792j$.

All poles have negative real parts, confirming stability of the linear augmented LQI design.

These gains were frozen before the final comparative validation so that later controller comparisons did not involve additional retuning.

9.11 Nonlinear Actuator Validation

The tuned LQI was subsequently evaluated using the same corrected longitudinal architecture used for the final PID comparison:

- restored thrust-control channel;
- forward-speed augmentation;
- first-order elevator actuator dynamics;
- elevator travel limits;
- elevator rate limit;
- identical 5° pitch command;
- identical reduced-elevator-effectiveness case.

Unlike the PID, the final LQI did not require the 0.10 s command shaper for actuator feasibility. The raw 5° step command was retained for the principal final LQI comparison.

For the nominal 5° command, the final LQI produced approximately:

- final pitch angle: 5.000°;
- overshoot: 0.682
- 2% settling time: 5.58 s;
- maximum requested elevator rate: 5.32°/s;
- maximum actual elevator rate: 5.32°/s;
- maximum elevator magnitude: 2.17°;
- maximum forward-speed deviation: 0.194
- maximum approximate angle-of-attack perturbation: 2.78°;
- maximum pitch rate: 1.92°/s.

All adopted transient and actuator criteria were satisfied.

The maximum elevator-rate demand used only approximately $\frac{5.32}{37} \times 100 \approx 14.4$ of the available rate capability. Thus, substantial actuator-rate margin remained during the nominal LQI maneuver.

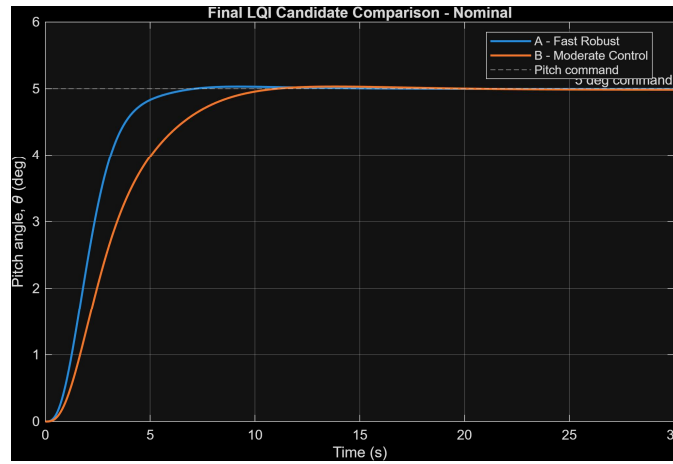


Figure 7. Nominal 5° pitch-command response of the final tuned LQI using the corrected longitudinal architecture and nonlinear elevator actuator. The controller achieves rapid tracking with approximately 0.68% overshoot and 5.58 s settling time while remaining well within the adopted actuator limits.

9.12 Reduced-Elevator-Effectiveness and Implementation Considerations

With elevator effectiveness reduced by 20%, the tuned LQI remained stable and continued to track the 5° pitch command.

The principal results were approximately:

- overshoot: 1.65
- settling time: 5.49 s;
- maximum requested elevator rate: 5.41°/s;
- maximum elevator magnitude: 2.60°;
- maximum forward-speed deviation: 0.195
- maximum approximate angle-of-attack perturbation: 2.79°.

The controller therefore retained large margins relative to the project overshoot, settling-time, and actuator-rate limits.

However, the LQI and PID architectures have different implementation requirements.

The PID primarily requires pitch-angle feedback, whereas the LQI control law requires the complete longitudinal state vector, $x = [u \ w \ q \ \theta]^T$.

A practical implementation would therefore require direct measurements, state estimation, or a combination of both.

No observer or state estimator was designed in this project.

The reported LQI performance must consequently be interpreted as the simulated performance of the full-state feedback law under the assumption that the required states are available accurately.

9.13 Engineering Interpretation

The advanced-control study demonstrates why controller comparison should include more than nominal pitch tracking.

The tuned LQI provided substantially lower overshoot, faster settling, and lower actuator-rate demand than the classical PID while also maintaining small internal-state excursions.

Its performance also remained strong under the 20% reduced-elevator-effectiveness case and throughout the later 500-trial numerical uncertainty study.

The improved results should not be attributed solely to the fact that LQI is an “optimal” controller. The two architectures use fundamentally different feedback information.

The PID principally reacts to pitch-angle tracking error and its integral and filtered derivative terms.

The LQI instead uses simultaneous feedback of u , w , q , and θ together with integrated pitch error.

It therefore has access to substantially richer information about the aircraft longitudinal state.

This added feedback capability provides stronger simulated performance but introduces additional implementation complexity because reliable state measurements or estimation would be required.

Accordingly, the final controller trade-off is not simply one of better versus worse control. The PID represents the lower-complexity classical architecture, while the LQI represents the higher-performance full-state architecture under the assumptions of the present simulation study.

10. Final PID versus Tuned-LQI Comparison

10.1 Objective

The final controller comparison evaluated the reference-shaped PID and tuned LQI using the corrected longitudinal architecture.

Both controllers used the same:

- Boeing 747 longitudinal aircraft model;
- restored thrust-control channel;
- forward-speed augmentation;
- nonlinear elevator actuator;
- elevator travel limits;
- $(37^\circ/\text{s})$ actuator-rate limit;
- nominal 5° pitch command;
- 20% reduced-elevator-effectiveness case.

The PID used the final first-order reference shaper with $\tau_r = 0.10$ s,

while the LQI was evaluated using the raw 5° step because its actuator demand remained comfortably below the adopted physical limit without command shaping.

The purpose of the comparison was to assess tracking performance, actuator demand, internal-state behavior, robustness, and implementation complexity rather than considering pitch response alone.

10.2 Final Quantitative Comparison

The principal corrected-architecture results are summarized below.

Table 7. Final PID and Tuned-LQI Performance Comparison

Metric	Final PID	Tuned LQI
Nominal overshoot	15.33%	0.682%
Nominal 2% settling time	14.07 s	5.58 s
Nominal maximum requested elevator rate	31.57°/s	5.32°/s
Nominal maximum elevator magnitude	4.68°	2.17°
Nominal maximum forward-speed deviation	0.227%	0.194%
Nominal maximum approximate $ \Delta\alpha $	3.84°	2.78°
Nominal maximum pitch rate	3.16°/s	1.92°/s
20% reduced-effectiveness overshoot	16.23%	1.65%
20% reduced-effectiveness settling time	15.09 s	5.49 s
20% reduced-effectiveness requested rate	31.58°/s	5.41°/s
Nominal project criteria	PASS	PASS
20% reduced-effectiveness criteria	PASS	PASS
500-trial stability/tracking	500/500	500/500
500-trial all-criteria result	475/500	500/500

Both controllers remained stable, tracked the nominal command, and satisfied the adopted actuator constraints.

The tuned LQI nevertheless achieved substantially stronger transient performance and required much less actuator motion.

10.3 Nominal Pitch-Command Tracking

For the nominal 5° command, the final PID produced $M_p = 15.33$ and $t_s = 14.07$ s.

The tuned LQI produced $M_p = 0.682$ and $t_s = 5.58$ s.

Relative to the PID, the LQI reduced nominal overshoot by approximately 95.55 and reduced settling time by approximately 60.34%

Both controllers converged to the commanded pitch with negligible final tracking error.

The primary difference therefore lies in transient performance rather than steady-state accuracy.

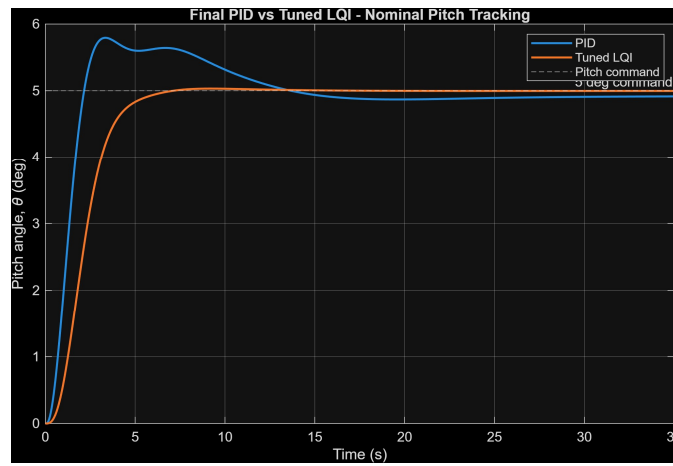


Figure 8. Final actuator-constrained PID and tuned-LQI responses to a 5° pitch command. The tuned LQI provides substantially lower overshoot and faster settling while both controllers converge to the commanded pitch.

10.4 Actuator-Rate Demand

A major difference between the controllers appeared in requested actuator-rate demand.

The reference-shaped PID required approximately $31.57^\circ/\text{s}$, which corresponds to approximately 85.3% of the adopted $37^\circ/\text{s}$ rate capability.

The tuned LQI required only approximately $5.32^\circ/\text{s}$, or about 14.4% of the same limit.

Relative to the PID, this represents an actuator-rate-demand reduction of approximately 83.14.

The LQI therefore achieved faster and more heavily damped pitch tracking while requiring substantially less rapid elevator motion.

This result is especially important because the improved LQI response was not obtained by driving the actuator harder.

10.5 Reduced Elevator Effectiveness

Both controllers were also evaluated with 20% reduced elevator effectiveness.

The final PID produced approximately $M_p = 16.23\%$ and $t_s = 15.09\text{ s}$.

The tuned LQI produced approximately $M_p = 1.65\%$ and $t_s = 5.49\text{ s}$.

Both controllers therefore satisfied the adopted project transient criteria.

Relative to the PID, the LQI reduced weak-elevator overshoot by approximately 89.83%.

and reduced weak-elevator settling time by approximately 63.59%.

Requested actuator rate was reduced from approximately $31.58^\circ/\text{s}$ for PID to $5.41^\circ/\text{s}$ for LQI, corresponding to an approximately 82.87% reduction.

The corrected architecture therefore changes an important conclusion from the earlier development study: the final PID no longer fails the weak-elevator settling-time requirement.

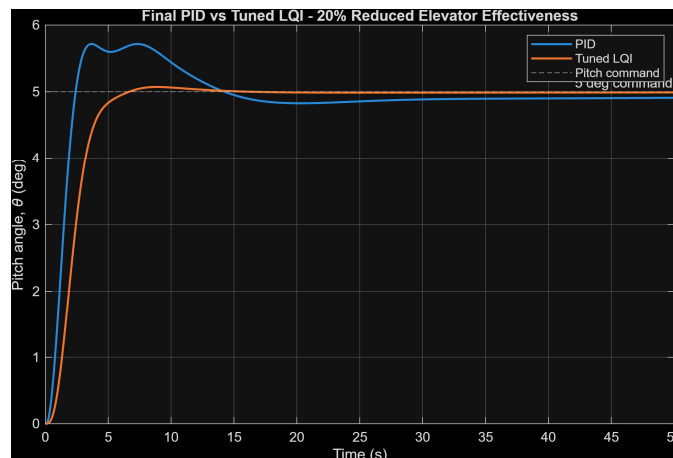


Figure 9. Final PID and tuned-LQI pitch responses with 20% reduced elevator effectiveness. Both controllers remain stable and satisfy the adopted transient criteria, while the tuned LQI retains substantially lower overshoot and faster settling.

10.6 Internal-State Comparison

Pitch angle alone does not fully describe the quality of the longitudinal aircraft response.

For the nominal 5° command, the PID produced approximately:

- maximum forward-speed perturbation: 0.535 m/s;
- maximum forward-speed deviation: 0.227% of trim speed;
- maximum approximate angle-of-attack perturbation: 3.84°;
- maximum pitch rate: 3.16°/s.

The tuned LQI produced approximately:

- maximum forward-speed perturbation: 0.457 m/s;
- maximum forward-speed deviation: 0.194% of trim speed;
- maximum approximate angle-of-attack perturbation: 2.78°;
- maximum pitch rate: 1.92°/s.

Both controllers therefore maintained small forward-speed departure from the trim condition.

The LQI also produced smaller vertical-velocity, approximate angle-of-attack, and pitch-rate excursions, providing additional evidence that its improved pitch response was not achieved by creating larger internal aircraft motions.

10.7 Numerical Uncertainty Comparison

The 500-trial numerical uncertainty study provided a broader comparison than the isolated nominal and weak-elevator cases.

All 500/500 PID trials remained stable and successfully tracked the commanded pitch. Of these, 475/500 satisfied every defined project criterion.

The 25 screening failures were caused only by overshoot exceeding the selected 20% threshold. All 500/500 LQI trials remained stable, tracked the command, and satisfied every defined project criterion.

The LQI also showed substantially tighter variation in overshoot, settling time, and actuator demand.

These results do not represent real-aircraft success probabilities because the coefficient distributions were selected numerical engineering perturbations rather than experimentally identified uncertainty statistics.

Within the tested perturbation set, however, the LQI demonstrated substantially more consistent transient performance.

10.8 Relative Performance Improvements

The principal final corrected-architecture improvements obtained with LQI relative to PID are summarized below.

Table 8. Relative Performance Improvements with Tuned LQI

Performance Quantity	Improvement with Tuned LQI
Nominal overshoot reduction	95.55%
Nominal settling-time reduction	60.34%
Nominal requested actuator-rate reduction	83.14%
Weak-elevator overshoot reduction	89.83%
Weak-elevator settling-time reduction	63.59%
Weak-elevator requested actuator-rate reduction	82.87%

These comparisons use the final corrected longitudinal architecture.

The developmental disturbance-rejection results from Section 7 are not included in this final percentage table because they were generated before the later thrust/airspeed and command-shaping modifications.

10.9 Classical Simplicity versus Advanced Performance

The quantitative results strongly favor the tuned LQI, but controller selection must also consider implementation complexity.

The PID has several practical advantages:

- familiar classical-control structure;
- relatively intuitive gain interpretation;
- principally pitch-based feedback;
- no requirement for full longitudinal-state feedback;
- successful nominal and reduced-effectiveness tracking;
- compliance with the adopted actuator constraints.

The tuned LQI provides:

- much lower overshoot;
- faster settling;
- substantially lower actuator-rate demand;
- smaller internal-state excursions;

- stronger performance under reduced elevator effectiveness;
- greater consistency in the numerical uncertainty study.

These benefits come at the cost of additional implementation requirements.

The LQI requires the complete longitudinal state vector $[u, w, q, \theta]^T$

together with the integral tracking state.

A real implementation would therefore require sufficiently accurate sensors and/or a state estimator.

No observer was designed in the present project.

The comparison is consequently a tradeoff between lower implementation complexity for PID and stronger simulated performance for LQI.

10.10 Final Controller Selection

Within the scope of the present simulation study, the tuned LQI is selected as the highest-performing controller.

It provided superior performance in:

- nominal overshoot;
- nominal settling time;
- actuator-rate demand;
- reduced-elevator-effectiveness tracking;
- internal-state excursions;
- consistency across the 500-trial numerical uncertainty study.

The final reference-shaped PID remains a credible lower-complexity classical benchmark. It remained stable and successfully tracked all 500 numerical uncertainty cases, satisfied the nominal and 20% reduced-elevator-effectiveness project criteria, and respected the adopted actuator constraints.

The controller hierarchy is therefore:

Reference-Shaped PID: Lower-Complexity Classical Benchmark
--

Tuned LQI: Highest-Performing Controller in the Present Simulation Study
--

The LQI selection should not be interpreted as universal superiority of LQI over PID. The conclusion applies to the present aircraft model, operating condition, actuator assumptions, controller structures, command cases, and numerical uncertainty model.

The next stage therefore performs final system-level validation of the frozen configurations and documents the additional robustness, internal-state, and frequency-domain evidence.

11. Final System Validation

11.1 Objective

After completion of controller development, actuator modeling, model-validity correction, reference shaping, robustness analysis, and advanced-control tuning, a final system-validation stage was performed using frozen controller configurations.

The purpose of this stage was to evaluate the completed control architectures without further gain modification.

The final validation considered:

- nominal 5° pitch-command tracking;
- a smaller 2° pitch command;
- a 5° ramp command;
- 20% reduced elevator effectiveness;
- actuator travel and rate demand;
- forward-speed departure;
- approximate angle-of-attack excursion;
- pitch-rate excursion;
- frequency-domain robustness;
- repeated numerical model uncertainty;
- consistency of the frozen controller configuration.

The developmental disturbance study from Section 7 was retained as supplementary evidence but was not treated as part of the principal corrected-architecture validation matrix.

11.2 Frozen Final Configuration

The final classical benchmark was the reference-shaped PID controller $K_p = 1$, $K_i = 0.5$, $K_d = 0.8$, with derivative-filter time constant $T_f = 0.1$ s and reference-shaper time constant $\tau_r = 0.10$ s.

The final advanced controller used $K_x = [-0.0004253 \quad 0.0022330 \quad -1.6776694 \quad -2.6675019]$, with $K_\xi = 1.2$.

Both controllers used the same corrected longitudinal model containing the elevator and thrust-control channels.

The supplementary thrust/airspeed augmentation was $\delta_p = \frac{\theta}{0.3} - K_u u$, with $K_u = 0.05$.

The modeled elevator actuator remained $G_{act}(s) = \frac{37}{s+37}$, with travel limits $-23^\circ \leq \delta_e \leq 17^\circ$ and actuator-rate limit $|\dot{\delta}_e| \leq 37^\circ/\text{s}$.

The frozen controller and actuator parameters were stored in `final_control_configuration.mat`.

No controller gains were modified during the final validation studies.

11.3 Multi-Command Validation

To avoid assessing the controllers using only one command amplitude and shape, the final architectures were evaluated using:

1. a 2° pitch step;
2. a 5° pitch step;
3. a 5° command reached through a 2 s ramp.

The principal results are summarized below.

Table 9. Final Multi-Command Validation Matrix

Controller	Command	Overshoot	2% Settling Time	Max Requested Elevator Rate	$Max \Delta u /U_0$	$Max \Delta \alpha $	Result
PID	2° step	15.33%	14.07 s	12.63°/s	0.091%	1.53°	PASS
LQI	2° step	0.682%	5.58 s	2.13°/s	0.078%	1.11°	PASS
PID	5° step	15.33%	14.07 s	31.57°/s	0.227%	3.84°	PASS
LQI	5° step	0.682%	5.58 s	5.32°/s	0.194%	2.78°	PASS
PID	5° / 2 s ramp	14.10%	13.12 s*	2.34°/s	0.225%	3.56°	PASS
LQI	5° / 2 s ramp	0.657%	4.69 s*	1.08°/s	0.192%	2.66°	PASS

** Settling time measured after the command reaches its final value.*

Both controllers successfully tracked all three command profiles.

The approximately proportional scaling between the 2° and 5° step responses provides additional evidence that the simulated motions remain consistent with the intended small-perturbation linear model.

The 2 s ramp command also illustrates the benefit of command shaping. Relative to the 5° step case, actuator-rate demand decreased substantially for both controllers while the final pitch remained accurately tracked.

11.4 Internal-State Validation

Final validation included explicit inspection of the longitudinal internal states rather than relying on pitch angle alone.

For the nominal 5° command, the PID produced approximately: $\max |\Delta u| = 0.535$ m/s, corresponding to $0.227\% U_0$, together with $\max |\Delta \alpha| \approx 3.84^\circ$ and $\max |q| \approx 3.16^\circ/\text{s}$.

The tuned LQI produced approximately: $\max |\Delta u| = 0.457$ m/s, or $0.194\% U_0$, with $\max |\Delta \alpha| \approx 2.78^\circ$ and $\max |q| \approx 1.92^\circ/\text{s}$.

The final architecture therefore keeps forward-speed departure small relative to the trim velocity while also maintaining moderate angle-of-attack and pitch-rate excursions.

The LQI produces smaller internal-state excursions than the PID for the nominal command.

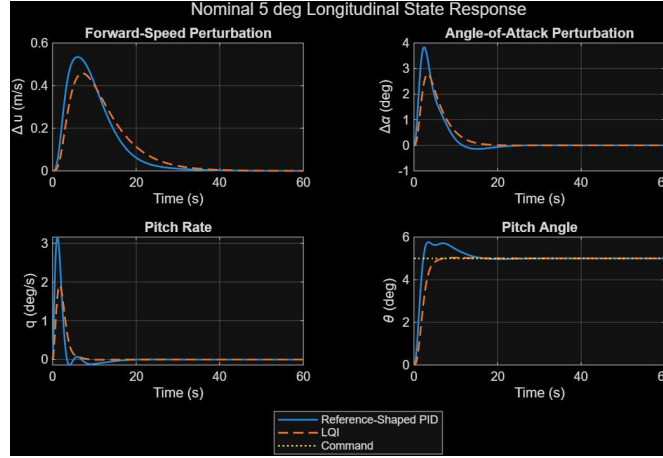


Figure 10. Nominal 5° command internal-state comparison for the final PID and tuned-LQI controllers. The responses show pitch angle together with forward-speed perturbation, approximate angle-of-attack perturbation, and pitch rate. Both controllers maintain small forward-speed departure from the trim condition, while the tuned LQI produces smaller internal-state excursions.

11.5 Frequency-Domain Robustness Assessment

Frequency-domain robustness was evaluated using linearized versions of the final feedback architectures.

For the PID analysis, the thrust/airspeed augmentation and elevator-actuator dynamics were included in the linearized feedback model. The nonlinear travel and rate saturations were excluded, as required for conventional linear frequency-response analysis. The external command shaper was also excluded from the feedback return ratio because it lies outside the feedback loop.

The final PID produced approximately: Phase Margin = 62.31° , Gain Margin = 32.24 dB, with gain-crossover frequency approximately $\omega_{gc} = 1.49$ rad/s.

The maximum sensitivity magnitude was $M_s \approx 1.18$.

The corresponding modulus margin was approximately 0.846.

These values indicate useful nominal linear robustness margin for the modeled PID loop.

For the full-state LQI architecture, a conventional single-input pitch-loop margin is not directly equivalent to the PID interpretation. Robustness was therefore evaluated using the actuator-channel return ratio of the augmented full-state feedback system.

The LQI actuator-channel analysis produced approximately: Phase Margin = 66.62° .

No finite gain-margin crossover was found over the evaluated frequency range, corresponding to an infinite reported gain margin for this particular return-ratio formulation.

The maximum sensitivity magnitude was approximately $M_s = 1.05$.

The result indicates strong linear robustness of the modeled actuator-feedback channel.

The LQI result should therefore be described specifically as an actuator-channel return-ratio robustness assessment rather than as a conventional SISO pitch-loop margin.

These margins are engineering screening results for the present linearized model and should not be interpreted as aircraft certification margins.

11.6 Final Robustness and Engineering Assessment

The final validation provides several independent layers of evidence.

Under the nominal 5° command, both controllers satisfied the project transient and actuator constraints.

With 20% reduced elevator effectiveness, the PID produced approximately 16.23% overshoot and 15.09 s settling time, while the LQI produced approximately 1.65% overshoot and 5.49 s settling time.

Both therefore passed the adopted reduced-elevator-effectiveness project criteria.

The 500-trial numerical uncertainty study provided a broader robustness test. Every PID and LQI trial remained stable and successfully tracked the pitch command.

The PID satisfied every defined screening criterion in 475/500 trials. The remaining 25 cases exceeded only the selected 20% overshoot threshold.

The LQI satisfied every defined screening criterion in 500/500 trials.

These counts should not be interpreted as real-aircraft probabilities because the uncertainty distributions were selected numerical perturbations rather than experimentally identified statistical distributions.

The final validation therefore supports the following engineering conclusions.

The reference-shaped PID is a stable, actuator-feasible, lower-complexity classical controller. It maintains small forward-speed departure, satisfies the nominal and reduced-elevator-effectiveness project criteria, and remains stable throughout the tested numerical uncertainty set.

The tuned LQI provides substantially stronger simulated transient performance, smaller internal-state excursions, lower actuator demand, and greater consistency across the tested uncertainty set.

The LQI remains subject to the important implementation assumption that the full longitudinal state vector is available for feedback. Accordingly, Reference-Shaped PID: Validated Classical Benchmark and Tuned LQI: Highest-Performing Controller in the Present Simulation Study.

12. Conclusions

12.1 Project Summary

This project developed and evaluated a longitudinal pitch-control system using a documented Boeing 747 small-perturbation model at Mach 0.8 and 40,000 ft.

The study progressed through:

- open-loop longitudinal mode identification;
- classical P, PI, and filtered-PID design;
- MATLAB–Simulink implementation validation;
- nonlinear elevator-actuator modeling;
- actuator travel and rate assessment;
- command shaping;
- restoration of the thrust-control channel for forward-speed validity;
- disturbance-rejection testing;
- full-state LQR/LQI development;
- reduced-control-effectiveness analysis;
- frequency-domain robustness assessment;
- multiple-command validation;
- internal-state validation;
- repeated numerical aerodynamic-uncertainty testing.

The principal engineering objective was not simply to obtain a fast pitch response, but to determine whether the resulting controller behavior remained dynamically stable, physically feasible, internally consistent with the small-perturbation model, and robust to selected modeling variations.

12.2 Open-Loop Aircraft Dynamics

The reconstructed Boeing 747 model exhibited the expected two principal longitudinal oscillatory modes.

The short-period poles were approximately $-0.3717 \pm 0.8869j$, corresponding to $\omega_n \approx 0.962$ rad/s and $\zeta \approx 0.387$.

The phugoid poles were approximately $-0.00329 \pm 0.06720j$, with $\omega_n \approx 0.0673$ rad/s and $\zeta \approx 0.0489$.

The phugoid was therefore stable but very lightly damped.

Long-duration elevator-only simulations initially produced extremely long mathematical settling behavior. However, later forward-speed auditing showed that such long-horizon responses could depart significantly from the original trim condition.

The open-loop response is therefore used primarily for modal validation rather than unrestricted long-duration physical interpretation.

12.3 Classical Controller Development

Proportional, proportional-integral, and filtered proportional-integral-derivative controllers were developed sequentially.

Proportional control improved response speed but retained steady-state tracking error.

Integral action eliminated steady-state error, while derivative feedback substantially improved transient behavior.

The filtered PID architecture $K_p = 1$, $K_i = 0.5$, $K_d = 0.8$, $T_f = 0.1$ s was selected as the final classical feedback structure.

The early ideal-actuator results were retained as controller-development evidence, but they were not treated as final aircraft-system performance.

12.4 Importance of Actuator Modeling and Command Shaping

Actuator modeling produced one of the most important design findings of the project.

The adopted Boeing 747 elevator actuator model was $G_{act}(s) = \frac{37}{s+37}$, with elevator travel limits of $-23^\circ \leq \delta_e \leq 17^\circ$ and maximum rate $|\dot{\delta}_e| \leq 37^\circ/\text{s}$.

The first-order actuator lag alone had only a modest effect on pitch response.

However, direct inspection of controller demand showed that the final PID feedback law, when subjected to an unshaped 5° step reference, requested approximately $185.1^\circ/\text{s}$, far above the modeled actuator-rate capability.

A first-order reference shaper was therefore introduced.

The selected value, $\tau_r = 0.10$ s, reduced nominal requested actuator rate to approximately $31.57^\circ/\text{s}$, allowing the physical actuator to follow the requested motion without sustained rate saturation.

This demonstrates that actuator feasibility must be assessed directly at the control-demand level rather than inferred from aircraft output response alone.

12.5 Forward-Speed Validity Correction

A later model-validity audit identified an important limitation in the original elevator-only architecture.

For a representative 5° command, the original configuration produced approximately $|\Delta u| \approx 36.7$ m/s after 60 s, corresponding to approximately 15.6% of the trim speed.

Such a large departure undermines the small-perturbation interpretation of the long-duration simulation.

The documented thrust-control channel was therefore restored, and a supplementary model-level forward-speed augmentation was introduced: $\delta_p = \frac{\theta}{0.3} - 0.05u$.

With this correction, the final nominal PID and LQI maneuvers produced maximum forward-speed departures of only approximately 0.227% and 0.194% of trim speed, respectively.

The thrust augmentation should be interpreted as a model-validity correction and simplified airspeed-hold representation. Detailed engine dynamics, thrust-command limits, and propulsion-system implementation were not modeled.

12.6 Final PID Performance

Using the corrected longitudinal architecture, nonlinear actuator model, and 0.10 s reference shaper, the final PID produced for the nominal 5° command: $M_p \approx 15.33\%$ and $t_s \approx 14.07$ s.

Maximum requested elevator rate was approximately 31.57°/s, remaining below the adopted 37°/s limit.

With 20% reduced elevator effectiveness, the final PID produced approximately 16.23% overshoot and 15.09 s settling time.

The final PID therefore satisfied the adopted nominal and reduced-effectiveness transient criteria.

The corrected architecture reverses the earlier developmental conclusion that the weak-elevator PID necessarily fails the 30 s settling-time screen.

12.7 Final LQI Performance

A conventional LQR regulator was first developed, but accurate nonzero pitch-command tracking required integral action.

The final tuned LQI used $K_x = [-0.0004253 \quad 0.0022330 \quad -1.6776694 \quad -2.6675019]$ and $K_\xi = 1.2$.

For the nominal 5° step command, the LQI achieved approximately $M_p = 0.682\%$ and $t_s = 5.58$ s, with requested elevator rate only 5.32°/s.

Under 20% reduced elevator effectiveness, the LQI retained approximately 1.65% overshoot and 5.49 s settling time.

The LQI also produced smaller pitch-rate, approximate angle-of-attack, and forward-speed excursions than the PID.

Its principal implementation limitation is that the control law assumes availability of the complete longitudinal state vector. No observer or estimator was developed in this project.

12.8 Robustness and Final PID–LQI Comparison

The final frequency-domain analysis indicated useful linear robustness margins for both controller architectures.

The PID produced approximately 62.31° phase margin and 32.24 dB gain margin, with $M_s \approx 1.18$.

The LQI actuator-channel return-ratio assessment produced approximately 66.62° phase margin and $M_s \approx 1.05$.

The LQI margin should be interpreted specifically as an actuator-channel full-state return-ratio result rather than a conventional SISO pitch-loop margin.

The 500-trial numerical uncertainty study further distinguished the controllers.

All 500/500 PID trials remained stable and tracked the commanded pitch, while 475/500 satisfied every defined project criterion.

The remaining 25 trials exceeded only the selected 20% overshoot threshold.

All 500/500 LQI trials remained stable, tracked the command, and satisfied every defined project criterion.

These results are not real-aircraft probabilities because the coefficient perturbations were selected engineering stress-test distributions rather than experimentally measured uncertainty statistics.

Within the tested numerical uncertainty set, however, the LQI demonstrated substantially greater performance consistency.

Relative to the final PID, the LQI reduced nominal: overshoot by approximately 95.55%, settling time by approximately 60.34%, and requested actuator-rate demand by approximately 83.14%.

The LQI therefore provided the strongest simulated performance of the investigated controller architectures.

12.9 Final Conclusion

The principal engineering conclusion of the project is that progressively increasing model realism materially changed the interpretation of controller performance.

An aircraft response that initially appeared acceptable required further examination once actuator limitations and internal aircraft states were inspected.

Actuator-rate analysis showed that acceptable pitch tracking could still demand physically excessive surface motion.

Forward-speed auditing showed that mathematically convergent long-duration results could still leave the intended linearization region.

Command shaping and thrust/airspeed augmentation corrected these weaknesses without requiring arbitrary controller retuning.

The final comparison therefore supports two distinct controller roles:

Reference-Shaped PID: Validated Lower-Complexity Classical Benchmark
--

Tuned LQI: Highest-Performing Controller in the Present Simulation Study
--

The PID remains attractive because of its simpler feedback architecture and successful performance under the final corrected model.

The LQI offers substantially stronger transient performance, lower actuator demand, smaller internal-state excursions, and greater consistency over the tested numerical uncertainty set, but requires full-state information.

The project therefore demonstrates a complete control-engineering workflow in which aircraft physics, actuator limitations, software validation, controller architecture, robustness, and model validity are evaluated together rather than treating command tracking alone as sufficient evidence of a successful design.

13. Limitations and Future Work

13.1 Linear Aircraft Model

The aircraft model used in this study is a linear small-perturbation representation developed about one specific cruise operating condition. Consequently, the results apply locally around the selected trim point and should not automatically be extended to the complete Boeing 747 flight envelope.

The model does not explicitly represent:

- stall,
- large-angle maneuvers,
- strongly nonlinear aerodynamic behavior,
- large changes in Mach number,
- large changes in altitude,
- nonlinear control-surface effectiveness.

The controller results should therefore be interpreted as a longitudinal control study near the modeled trim condition.

13.2 Longitudinal Dynamics Only

Only longitudinal aircraft motion was considered. The modeled states were $\mathbf{x}_{long} = [u \ w \ q \ \theta]^T$. Lateral-directional dynamics such as $\mathbf{x}_{lat-dir} = [v \ p \ r \ \phi \ \psi]^T$ were outside the scope of the project. Coupling between longitudinal and lateral-directional aircraft motion was therefore not evaluated. A future extension could employ a complete six-degree-of-freedom aircraft model.

13.3 Rigid-Aircraft Assumption

The aircraft was treated as a rigid body. Structural flexibility and aeroelastic effects were not included in the flight-control model. Consequently, interactions between the pitch-control system and flexible structural modes were not evaluated. For a higher-fidelity study, flexible-body dynamics could be included before evaluating controller bandwidth and stability margins.

13.4 Actuator and Thrust-Augmentation Limitations

The baseline longitudinal aircraft dynamics and elevator-actuator constraints were obtained from separate documented Boeing 747 sources.

The elevator actuator was adopted as a representative Boeing 747-100/200 benchmark, $G_{act}(s) = \frac{37}{s+37}$, with travel limits $-23^\circ \leq \delta_e \leq 17^\circ$ and rate limit $|\dot{\delta}_e| \leq 37^\circ/\text{s}$.

The project does not claim that the baseline state-space model and actuator source correspond to the exact same Boeing 747 configuration, loading condition, or trim point.

The final longitudinal architecture also uses the restored model thrust-control channel with the supplementary relation $\delta_p = \frac{\theta}{0.3} - 0.05u$ to prevent excessive forward-speed departure during pitch maneuvers.

This thrust augmentation is a simplified model-level airspeed-validity correction. Detailed engine dynamics, propulsion-system lag, thrust-command saturation, engine spool limits, and autothrottle implementation were not modeled.

The thrust-control perturbation should therefore not be interpreted directly as a physical throttle-percentage command.

A higher-fidelity study should use a configuration-matched aircraft, actuator, and propulsion model with documented control-input definitions and physical propulsion limits.

13.5 Uncertainty-Study Limitation

The final uncertainty assessment used 500 reproducible numerical trials in which selected nonzero aerodynamic coefficients of the longitudinal state matrix and elevator-input vector were independently perturbed by uniformly distributed variations of $\pm 10\%$.

The gravity and kinematic matrix entries were excluded from these perturbations.

This approach provides a broader robustness assessment than the earlier isolated parameter sweeps, but the assumed variations are engineering stress-test inputs rather than experimentally identified Boeing 747 uncertainty statistics.

The study does not explicitly include:

- measured covariance between aerodynamic derivatives;
- configuration-specific flight-test uncertainty;
- probabilistic center-of-gravity variation;
- correlated Mach-number and altitude variation;
- mass and inertia uncertainty;
- formal structured uncertainty models.

Consequently, results such as 475/500 PID cases and 500/500 LQI cases satisfying the defined project criteria should not be interpreted as real-aircraft success probabilities.

They describe controller performance only over the selected reproducible numerical perturbation set.

Future robustness work could use experimentally justified uncertainty bounds, correlated parameter distributions, structured singular-value analysis, and other formal robust-control methods.

13.6 Disturbance-Model Limitation

The disturbance model used in this study was an additive control-channel disturbance. It provided a controlled method for comparing feedback disturbance rejection, but it does not represent the complete physical disturbance environment of an aircraft.

The simulations did not explicitly model:

- atmospheric turbulence,
- vertical gusts,
- wind shear,
- sensor disturbances,
- structural excitation,
- engine disturbances.

Future work could introduce aerodynamic gust and turbulence models directly into the aircraft force and moment equations.

13.7 Full-State Feedback Assumption

The tuned LQI controller assumes availability of the complete longitudinal state vector: $x = [u \ w \ q \ \theta]^T$. No state observer or estimator was developed in the present project. A practical implementation would therefore require appropriate sensors and/or estimated states.

A logical next extension would be development of a state observer, such as:

- a Luenberger observer,
- a Kalman filter,
- another suitable state-estimation architecture.

The resulting controller-estimator combination could then be tested with sensor noise and model uncertainty.

13.8 Sensor and Computational Effects

The final simulations did not include a detailed model of:

- sensor noise,
- sensor bias,
- sensor bandwidth,
- data-sampling effects,
- computational delay,
- communication delay,

- digital controller discretization.

These effects can influence practical flight-control performance, particularly for higher-bandwidth state-feedback controllers. Future work should therefore evaluate the tuned LQI after introducing realistic measurement and implementation effects.

13.9 Gain Scheduling and Flight-Envelope Extension

The controller gains were designed for one operating condition. An aircraft operating over a broad range of altitude, speed, mass, and center-of-gravity conditions will experience changing dynamics. A more complete flight-control architecture could therefore develop controllers at multiple trim conditions and interpolate between them using gain scheduling. This would allow the control law to address a broader portion of the aircraft flight envelope.

13.10 Nonlinear and Six-Degree-of-Freedom Validation

A major future development would be migration from the current linear longitudinal plant to a nonlinear six-degree-of-freedom simulation.

The current controller could then be tested under:

- larger pitch commands,
- simultaneous longitudinal and lateral motion,
- nonlinear aerodynamic coefficients,
- changing airspeed and altitude,
- larger disturbance amplitudes,
- more realistic atmospheric conditions.

This would provide a stronger bridge between the present control-design study and a high-fidelity aircraft simulation.

13.11 Hardware-in-the-Loop and Experimental Validation

The present project is simulation-based. No hardware-in-the-loop or flight-test validation was performed. A future implementation could deploy the control algorithm on real-time hardware and connect it to a real-time aircraft simulation.

Such testing could evaluate:

- sampling effects,
- computational latency,
- actuator-command interfaces,
- sensor interfaces,
- numerical execution time,

- controller behavior under real-time constraints.

Hardware-in-the-loop testing would provide a practical intermediate step between desktop simulation and physical flight testing.

13.12 Recommended Future Development Path

A logical continuation of the project would proceed through progressively higher levels of implementation and aircraft-model fidelity.

The highest-priority next step is development of a state observer or Kalman filter for the LQI architecture. This would remove the present assumption that every longitudinal state is directly and accurately available for feedback.

The controller-estimator system could then be evaluated with realistic sensor noise, bias, bandwidth, sampling, and computational delay.

A subsequent aircraft-model extension should evaluate multiple documented trim conditions covering changes in airspeed, altitude, mass, and center-of-gravity location. These models could support gain scheduling or another flight-envelope control architecture.

The simplified thrust/airspeed augmentation should also be replaced with a documented propulsion model containing engine dynamics and physical command limits.

The resulting controller could then be transferred to a nonlinear six-degree-of-freedom aircraft simulation containing realistic aerodynamic disturbances and longitudinal–lateral coupling.

Finally, hardware-in-the-loop implementation could be used to evaluate real-time execution, sensor interfaces, actuator-command interfaces, numerical latency, and controller behavior under embedded-computing constraints.

A suitable future-development sequence is therefore:

State Estimation → Sensor and Digital-Implementation Effects → Multiple Flight Conditions and Propulsion Dynamics → Gain-Scheduled / Flight-Envelope control → Nonlinear 6-DOF Validation → Hardware-in-the-Loop Validation

These extensions were not completed in the present project and are proposed as future development opportunities.

References

- [1] J. P. How, “Aircraft Longitudinal Dynamics,” Lecture 6, 16.333 Aircraft Stability and Control, Massachusetts Institute of Technology OpenCourseWare, Cambridge, MA, USA, Fall 2004.
- [2] J. P. How, “Basic Longitudinal Control,” Lecture 9, 16.333 Aircraft Stability and Control, Massachusetts Institute of Technology OpenCourseWare, Cambridge, MA, USA, Fall 2004.
- [3] R. K. Heffley and W. F. Jewell, *Aircraft Handling Qualities Data*, NASA CR-2144, Systems Technology, Inc., Hawthorne, CA, USA, 1972.
- [4] J. Y. Shin, “Robust Gain-Scheduled Fault Tolerant Control for a Transport Aircraft,” NASA Technical Reports Server, Document ID 20070032920, 2007.
- [5] The MathWorks, Inc., *MATLAB Documentation*, Natick, MA, USA.
- [6] The MathWorks, Inc., *Simulink Documentation*, Natick, MA, USA.
- [7] The MathWorks, Inc., *Control System Toolbox Documentation*, Natick, MA, USA.

Appendices

Appendix A — Baseline Aircraft Model

The frozen Boeing 747 longitudinal small-perturbation model uses the state vector $x = [u \ w \ q \ \theta]^T$, where:

- u is forward-velocity perturbation, m/s;
- w is vertical body-axis velocity perturbation, m/s;
- q is pitch-rate perturbation, rad/s;
- θ is pitch-angle perturbation, rad.

The validated baseline state matrix is approximately $A = \begin{bmatrix} -0.0069 & 0.0139 & 0 & -9.81 \\ -0.0905 & -0.3149 & 235.8928 & 0 \\ 0.0004 & -0.0034 & -0.4282 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

The documented elevator-input vector is approximately $B_e = \begin{bmatrix} -5.726 \times 10^{-5} \\ -5.508 \\ -1.157 \\ 0 \end{bmatrix}$.

The restored thrust-control input used during the final model-validity studies is $B_p = \begin{bmatrix} 2.943 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

The complete final input structure is therefore $B = [B_e \ B_p]$.

Pitch angle is extracted using $C_\theta = [0 \ 0 \ 0 \ 1]$, $D_\theta = 0$.

The baseline aircraft corresponds to the documented Mach 0.8, 40,000 ft cruise condition used throughout the project, with trim speed approximately $U_0 = 235.9$ m/s.

The validated baseline plant is stored as B747_longitudinal_baseline.mat.

The original frozen baseline retains the documented aircraft model. The later thrust/airspeed augmentation is applied externally during final-system simulation rather than redefining the underlying aerodynamic state matrix.

Appendix B — Final Controller and Actuator Configuration

B.1 Final Reference-Shaped PID

The final classical controller uses $K_p = 1$, $K_i = 0.5$, $K_d = 0.8$, with derivative-filter time constant $T_f = 0.1$ s.

The final PID architecture also includes the first-order reference shaper $\tau_r \dot{\theta}_{cmd,f} + \theta_{cmd,f} = \theta_{cmd}$, with $\tau_r = 0.10$ s.

The shaped command $\theta_{cmd,f}$ is supplied to the PID tracking loop.

This reference shaper reduces nominal requested elevator-rate demand from approximately $185.1^\circ/\text{s}$ for the unshaped final PID command to approximately $31.57^\circ/\text{s}$, which is below the adopted actuator-rate limit.

B.2 Final Tuned LQI

The final tuned LQI controller uses the full longitudinal state vector and an integral pitch-error state.

The frozen state-feedback gain is $K_x = [-0.0004253003 \ 0.0022330081 \ -1.677669374 \ -2.667501937]$, with integral gain $K_\xi = 1.2$.

The implemented elevator-control law is $\delta_e = -K_x x - K_\xi \xi$, where $\dot{\xi} = \theta - \theta_{cmd}$.

The LQI uses the raw reference command in the final principal comparison because its nominal requested actuator rate remains substantially below the modeled physical rate limit without additional command shaping.

No state observer or estimator is included. The final LQI simulations therefore assume that the complete longitudinal state vector is available for feedback.

B.3 Final Elevator Actuator Model

Both final controllers use the same first-order elevator actuator: $G_{act}(s) = \frac{37}{s+37}$.

The adopted physical travel limits are $-23^\circ \leq \delta_e \leq 17^\circ$, with maximum rate $|\dot{\delta}_e| \leq 37^\circ/\text{s}$.

Requested and delivered elevator motion are monitored separately so that actuator feasibility can be assessed directly.

B.4 Thrust/Airspeed Augmentation

The final corrected longitudinal simulations retain the documented thrust-control input and apply the supplementary model-level relation $\delta_p = \frac{\theta}{0.3} - K_u u$, where $K_u = 0.05$.

The first term provides the steady feedforward contribution required to counter the forward-speed acceleration associated with a nonzero pitch perturbation in the linear model.

The second term provides supplementary feedback from the forward-speed perturbation.

This augmentation reduces excessive long-duration forward-speed departure and keeps the final nominal pitch maneuvers close to the original trim condition.

The variable δ_p is treated as the normalized thrust-control perturbation defined by the aircraft model. It should not be interpreted directly as physical throttle percentage.

Detailed engine dynamics, thrust-command saturation, propulsion lag, and autothrottle implementation are outside the scope of the project.

B.5 Frozen Configuration Files

The principal frozen configuration is stored as `final_control_configuration.mat`.

The validated aircraft model is stored separately as `B747_longitudinal_baseline.mat`.

The separation between baseline aircraft data and controller configuration preserves the original validated plant while allowing the final controller, actuator, reference-shaping, and supplementary thrust/airspeed architecture to be reconstructed explicitly during final analysis.

Appendix C — Key MATLAB and Simulink Files

The principal MATLAB and Simulink files used during aircraft modeling, controller development, validation, and final engineering assessment are summarized below.

Aircraft Modeling and Open-Loop Analysis

- `phase4_b747_model.m`
- `phase5_open_loop_modes.m`
- `phase5_open_loop_response.m`
- `phase6_matlab_simulink_validation.m`
- `phase6_b747_baseline.slx`

Classical Controller Development

- `phase7_p_final.m`
- `phase7_pi_final.m`

- phase7_pid_final.m
- phase7_controller_comparison.m
- phase7_pid_matlab_simulink_validation.m
- phase7_b747_pid_closed_loop.slx

Actuator and Disturbance Analysis

- phase10_actuator_dynamics.m
- phase10_actuator_limit_check.m
- phase10_constrained_controller_comparison.m
- phase10_final_constrained_controller.m
- phase10_constrained_disturbance_2deg.m
- phase10_b747_actuator_limits.slx

Development-Stage Sensitivity Analysis

- phase11_elevator_effectiveness_sensitivity.m
- phase11_weak_elevator_constrained.m
- phase11_pitch_damping_sensitivity.m
- phase11_combined_uncertainty.m

These Phase 11 scripts are retained as developmental sensitivity studies. The final uncertainty conclusions are based on the later corrected-architecture Phase 14 analysis.

Advanced Controller Development

- phase12_lqr_controllability.m
- phase12_lqr_initial_design.m
- phase12_lqi_initial_design.m
- phase12_lqi_weight_grid.m
- phase12_lqi_final_candidate_validation.m
- phase12_final_pid_lqi_comparison.m
- phase12_b747_lqi_actuator_limits.slx

Original Frozen-Configuration Validation

- phase13_freeze_final_configuration.m

- phase13_master_validation.m
- phase13_consistency_audit.m

These Phase 13 files document the original frozen-controller validation stage. Their numerical results were subsequently superseded where necessary by the corrected Phase 14 architecture.

Final Model-Validity and Controller Assessment

- phase14A_two_input_plant.m
- Phase 14A thrust/airspeed compensation analysis scripts
- Phase 14B reference-shaping sweep and final reference-shaper validation scripts
- Phase 14C corrected PID–LQI comparison scripts
- Phase 14D PID frequency-domain robustness analysis
- Phase 14D LQI actuator-channel return-ratio analysis
- phase14E_command_validation_matrix.m
- phase14E_internal_state_figures.m
- Phase 14E 500-trial numerical uncertainty analysis

The Phase 14 files form the principal basis for the final corrected-architecture results presented in Sections 8–12.

Principal Data Files

- B747_longitudinal_baseline.mat
- final_control_configuration.mat
- phase14E_command_validation_results.mat

The baseline aircraft and controller configuration are stored separately so that the validated aerodynamic plant is not modified implicitly during subsequent controller studies.

The Phase 14 command-validation results file stores the final multi-command simulation outputs used for the internal-state comparison and related final-validation figures.

Appendix D — Final Corrected-Architecture Validation Record

The original Phase 13 numerical consistency audit was completed before the later model-validity corrections, reference shaping, thrust/airspeed augmentation, and expanded robustness studies.

Those Phase 13 results remain part of the project-development history but no longer represent the final aircraft-control architecture.

The final report therefore uses the corrected Phase 14 results summarized in this appendix.

D.1 Final Nominal and Reduced-Effectiveness Results

Table 10. Final Corrected-Architecture Validation Summary

Quantity	PID	Tuned LQI
Nominal overshoot	15.33%	0.682%
Nominal 2% settling time	14.07 s	5.58 s
Nominal requested elevator rate	31.57°/s	5.32°/s
Nominal maximum elevator magnitude	4.68°	2.17°
Nominal maximum forward-speed deviation	0.227%	0.194%
Nominal maximum approximate $\Delta\alpha$	3.84°	2.78°
Nominal maximum pitch rate	3.16°/s	1.92°/s
20% reduced-effectiveness overshoot	16.23%	1.65%
20% reduced-effectiveness settling time	15.09 s	5.49 s
20% reduced-effectiveness requested elevator rate	31.58°/s	5.41°/s
Nominal project criteria	PASS	PASS
Reduced-effectiveness project criteria	PASS	PASS

These values correspond to the final corrected longitudinal architecture. The PID includes the 0.10 s reference shaper, whereas the LQI uses the unshaped step reference for the principal comparison.

D.2 Multi-Command Verification

The final controller configurations were evaluated using a 2° step, a 5° step, and a 5° command reached through a 2 s ramp.

Both controllers successfully tracked all three command profiles.

The approximately proportional response scaling between the 2° and 5° step cases supports consistency with the intended small-perturbation model.

The ramp-command results also confirmed that command shaping can substantially reduce actuator-rate demand while retaining accurate final pitch tracking.

The complete numerical results are reported in Table 10 of Section 11.

D.3 Internal-State Verification

For the nominal 5° command, the final PID produced approximately $\max |\Delta u| = 0.535$ m/s, $\max |\Delta\alpha| = 3.84^\circ$, and $\max |q| = 3.16^\circ/\text{s}$.

The tuned LQI produced approximately $\max |\Delta u| = 0.457$ m/s, $\max |\Delta\alpha| = 2.78^\circ$, and $\max |q| = 1.92^\circ/\text{s}$.

Relative to the trim speed of approximately 235.9 m/s, maximum forward-speed departure remained below 0.3% for both nominal controller cases.

This provides an important consistency check that the final commanded maneuvers remain much closer to the original linearization condition than the earlier elevator-only long-duration simulations.

D.4 Frequency-Domain Verification

The final PID linearized feedback architecture produced approximately Phase Margin = 62.31° , Gain Margin = 32.24 dB, and $M_s \approx 1.18$.

For the tuned LQI, the actuator-channel return-ratio assessment produced approximately Phase Margin = 66.62° and $M_s \approx 1.05$.

No finite gain-margin crossover was identified for the evaluated LQI actuator-channel return ratio.

The two margin calculations have different loop interpretations and should therefore not be treated as identical SISO robustness measures.

D.5 Numerical Uncertainty Verification

A reproducible 500-trial numerical uncertainty study was performed using independent uniform $\pm 10\%$ perturbations of selected aerodynamic coefficients.

For the PID: 500/500 trials remained stable and tracked the command, while 475/500 satisfied every defined project criterion.

The 25 remaining trials exceeded only the selected 20% overshoot criterion.

For the tuned LQI: 500/500 trials remained stable, tracked the command, and satisfied every defined project criterion.

These trial counts describe performance over the selected numerical perturbation set and are not interpreted as statistical probabilities for the physical aircraft.

D.6 Final Configuration Verification

The final report uses the following frozen controller values.

For the PID: $K_p = 1$, $K_i = 0.5$, $K_d = 0.8$, $T_f = 0.1$ s, $\tau_r = 0.10$ s.

For the tuned LQI: $K_x = [-0.0004253003 \quad 0.0022330081 \quad -1.677669374 \quad -2.667501937]$, $K_\xi = 1.2$.

The elevator actuator is $G_{act}(s) = \frac{37}{s+37}$, with $-23^\circ \leq \delta_e \leq 17^\circ$ and $|\dot{\delta}_e| \leq 37^\circ/\text{s}$.

The supplementary airspeed-validity architecture uses $\delta_p = \frac{\theta}{0.3} - 0.05u$.

The validated aircraft model remains stored separately from the controller configuration so that the original frozen aerodynamic plant is preserved.

D.7 Final Validation Status

The final corrected-architecture results support the following status:

Reference-Shaped PID: VALIDATED CLASSICAL BENCHMARK

Tuned LQI: VALIDATED HIGHEST-PERFORMING SIMULATED CONTROLLER

These conclusions apply only within the assumptions and limitations documented in the report.